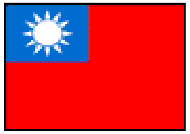
# Elliptic flow, non-flow and initial-state fluctuations at RHIC

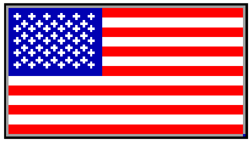
Constantin Loizides
(LBNL)

INT-10-2A: Opening workshop May 24-28, 2010
New results from LHC and RHIC

# The PHOBOS collaboration







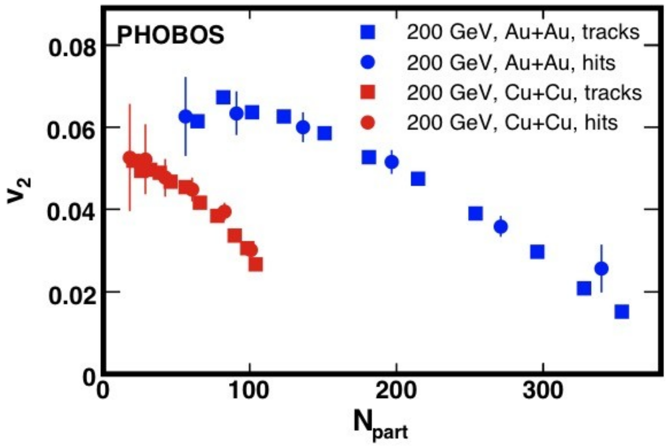
Burak Alver, Birger Back, Mark Baker, Maarten Ballintijn, Donald Barton, Russell Betts, Richard Bindel, Wit Busza (Spokesperson), Vasundhara Chetluru, Edmundo García, Tomasz Gburek, Joshua Hamblen, Conor Henderson, David Hofman, Richard Hollis, Roman Hołyński, Burt Holzman, Aneta Iordanova, Chia Ming Kuo, Wei Li, Willis Lin, Constantin Loizides, Steven Manly, Alice Mignerey, Gerrit van Nieuwenhuizen, Rachid Nouicer, Andrzej Olszewski, Robert Pak, Corey Reed, Christof Roland, Gunther Roland, Joe Sagerer, Peter Steinberg, George Stephans, Andrei Sukhanov, Marguerite Belt Tonjes, Adam Trzupek, Sergei Vaurynovich, Robin Verdier, Gábor Veres, Peter Walters, Edward Wenger, Frank Wolfs, Barbara Wosiek, Krzysztof Woźniak, Bolek Wysłouch

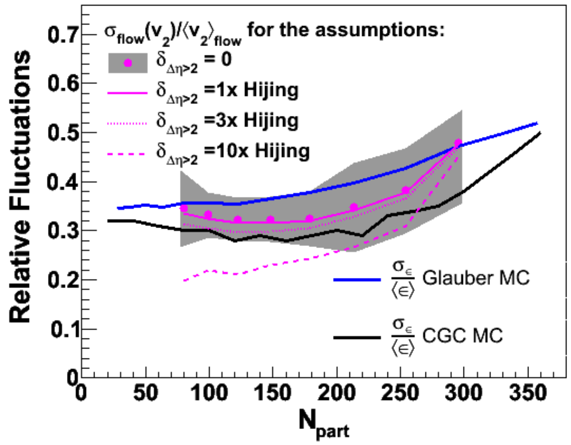
ARGONNE NATIONAL LABORATORY
INSTITUTE OF NUCLEAR PHYSICS PAN, KRAKOW
NATIONAL CENTRAL UNIVERSITY, TAIWAN
UNIVERSITY OF MARYLAND

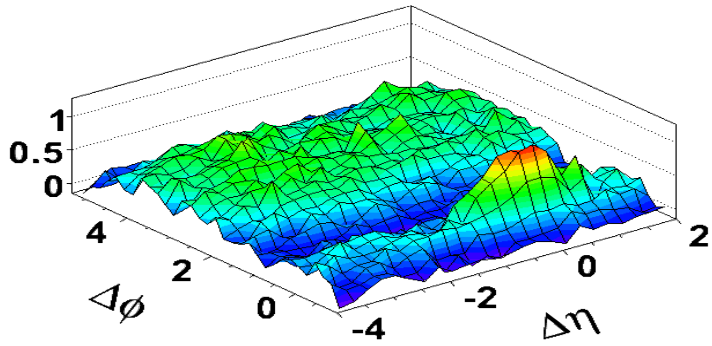
BROOKHAVEN NATIONAL LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
UNIVERSITY OF ILLINOIS AT CHICAGO
UNIVERSITY OF ROCHESTER

# Outline



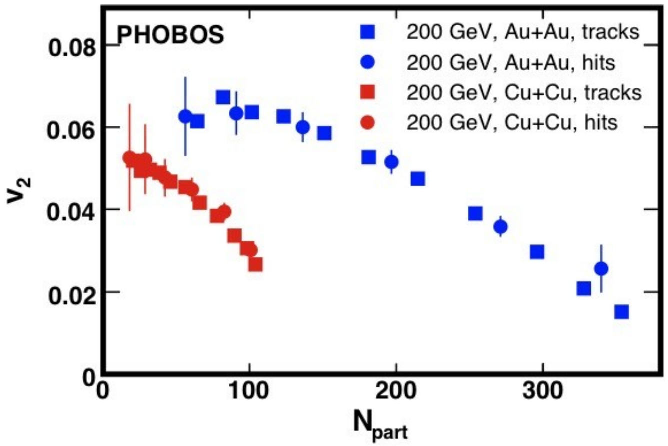


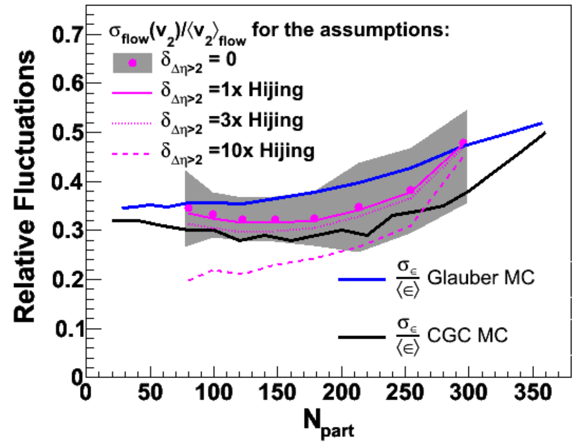



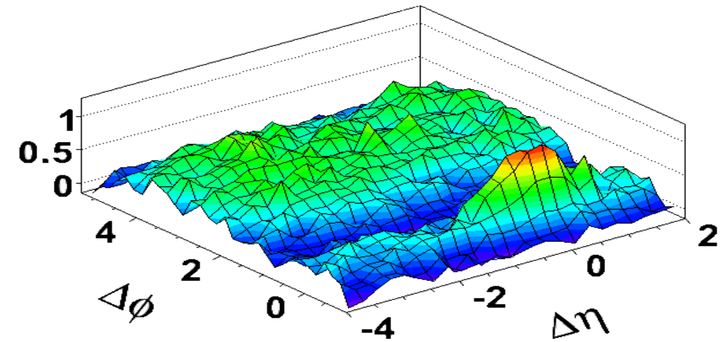




1) System size, energy, pseudorapidity, and centrality dependence of elliptic flow, PHOBOS, PRL 98, 242302, 2007 (nucl-ex/0610037)
2) Cluster properties from two-particle angular correlations in p + p collisions at 200 and 410-GeV, PHOBOS, PRC 75, 054913, 2007 (arXiv:0704.0966 [nucl-ex])
3) Importance of correlations and fluctuations on the initial source eccentricity in A+A collisions, PHOBOS + U.Heinz, PRC 77, 014906, 2008 (arXiv:0711.3724 [nucl-ex])
4) System size dependence of cluster properties from two-particle angular correlations in Cu+Cu and Au+Au collisions at 200 GeV, PRC 81, 024904, 2010 (arXiv:0812.1172 [nucl-ex])
5) High transverse momentum triggered correlations over a large pseudorapidity acceptance in Au+Au collisions at 200 GeV, PRL 104, 062301, 2010 (arXiv:0903.2811 [nucl-ex])
6) Event-by-event fluctuations of azimuthal particle anisotropy in Au + Au Collisions at 200 GeV, PRL, 104:142301, 2010 (nucl-ex/0702036)
7) Non-flow correlations and elliptic flow fluctuations in gold-gold collisions at 200 GeV, PRC81, 034915, 2010 (arXiv:1002.0534 [nucl-ex])
8) Collision geometry fluctuations and triangular flow in heavy-ion collisions, B. Alver, G. Roland, accepted in PRC, 2010 (arXiv:1003.0194 [nucl-th])

# Outline




1) System size, energy, pseudorapidity, and centrality dependence of elliptic flow,
PHOBOS, PRL 98, 242302, 2007 (nucl-ex/0610037)

2) Cluster properties from ... at 200 and 410-GeV,
PHOBOS, PRC 75, 054...

3) Importance of correlatio... in A+A collisions,
PHOBOS + U.Heinz, PI...

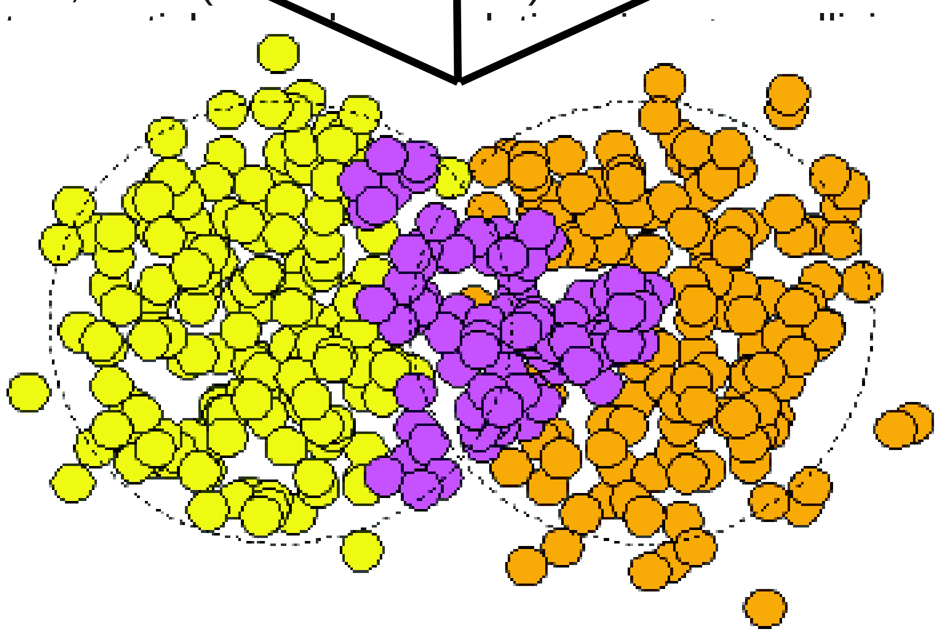
4) System size dependenc... rrelations
in Cu+Cu and Au+Au c... 0812.1172 [nucl-ex])

5) High transverse mome... dity acceptance in Au+Au
collisions at 200 GeV, P...

6) Event-by-event fluctuat... lisions at 200 GeV,
PRL, 104:142301, 2010...

7) Non-flow correlations a... 200 GeV,
PRC81, 034915, 2010 (...

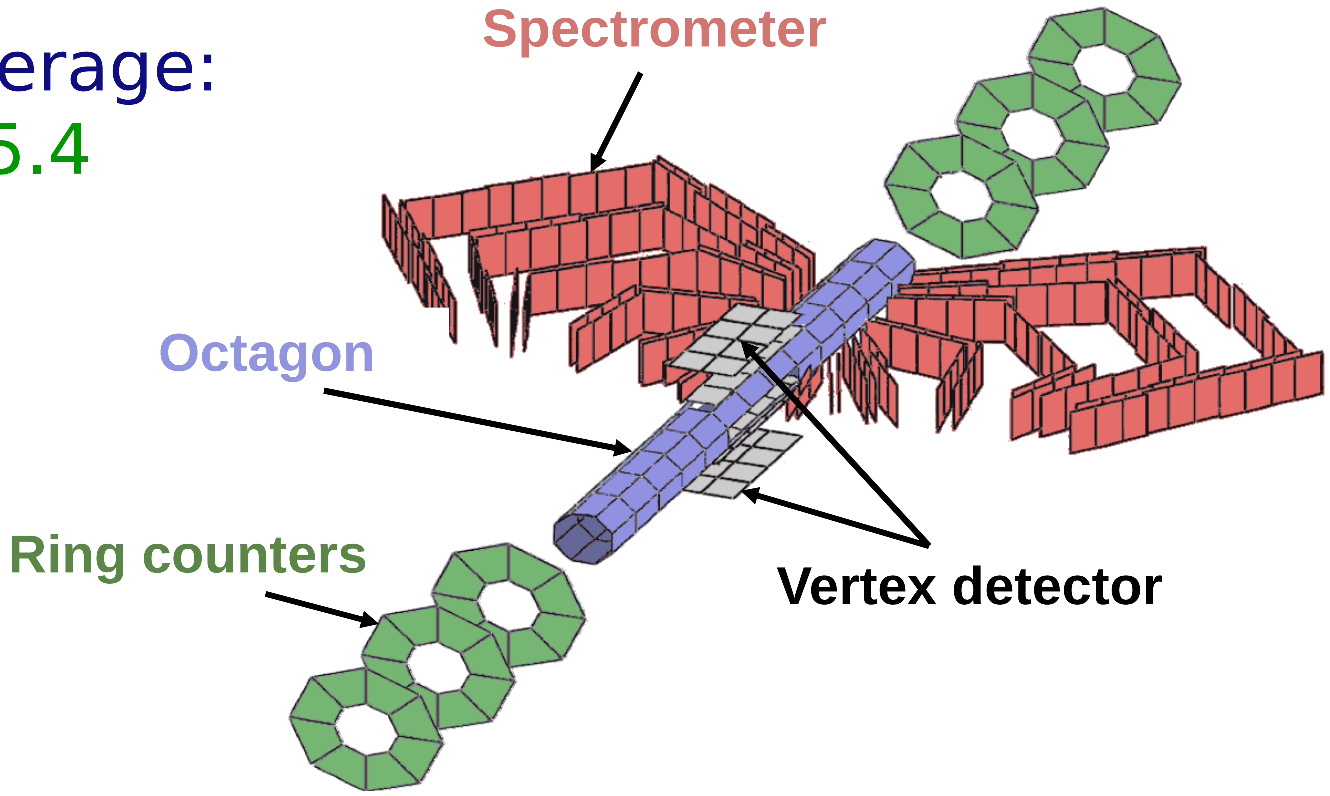
8) Collision geometry fluct...
B. Alver, G. Roland, accepted in PRC, 2010 (arXiv:1003.0194 [nucl-th])



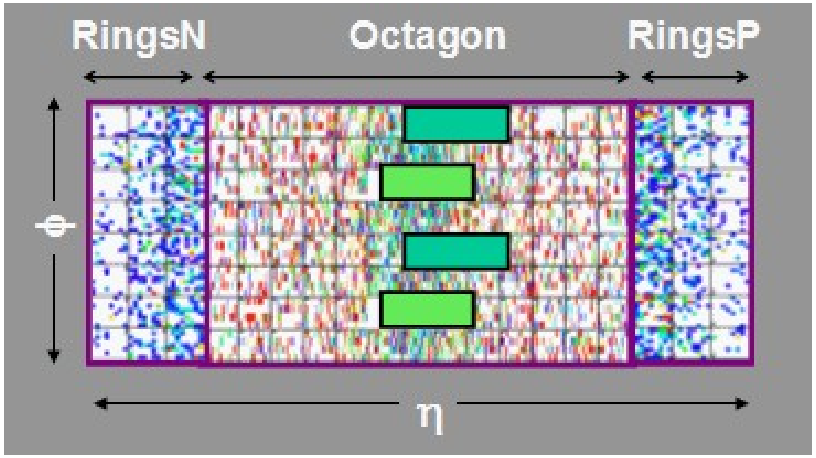
Burak Alver
next week

# Correlation studies with PHOBOS 

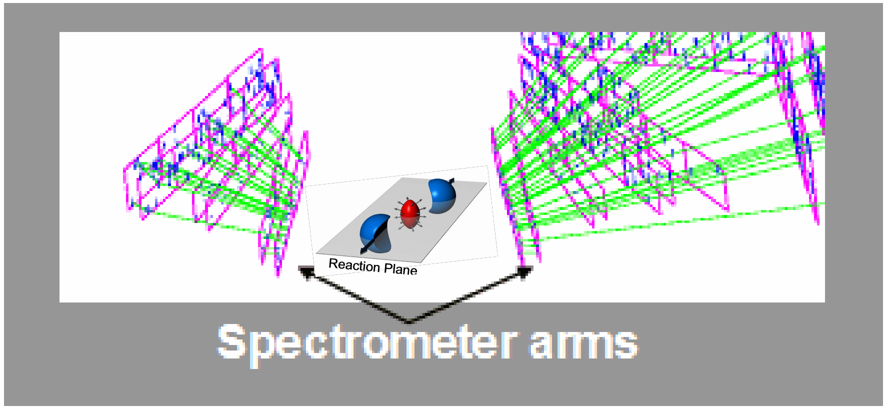
Large coverage:
$|\eta| < 5.4$




Hits (no $p_T$ information)

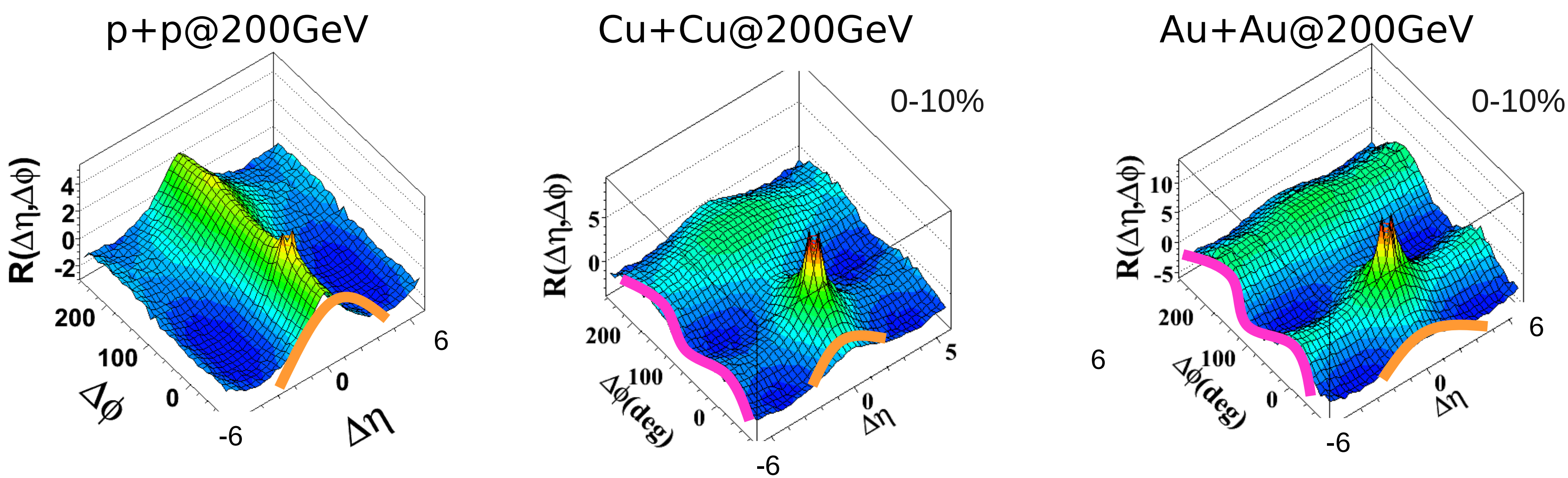



Tracks (near mid-rapidity)




(Dep. on vertex and trk quality: $0<\eta<2$)

# Inclusive two particle correlations



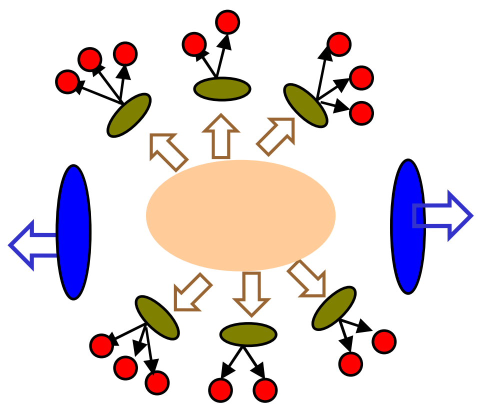

$$R(\Delta\phi,\Delta\eta)=\left\langle (n-1)\left(\frac{F_n(\Delta\phi,\Delta\eta)}{B_n(\Delta\phi,\Delta\eta)}-1\right)\right\rangle$$

Similar structure in p+p, Cu+Cu and Au+Au collisions,
a clear signal of elliptic flow is visible in A+A collisions

Remember: No $p_T$ cutoff (i.e. $p_T$>7 – 35 MeV/c)

PHOBOS, PRC 75 054913 (2007)
PHOBOS, PRC 81 024904 (2010)

# Cluster model fit to correlation in Δη 



## Cluster model:

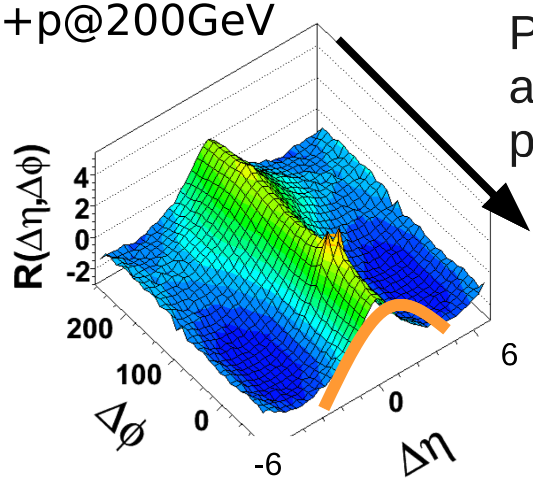
$$R(\Delta\eta)=(k_{\text{eff}}-1)\left(\frac{G(\Delta\eta)}{B(\Delta\eta)}-1\right) \qquad G(\Delta\eta)\simeq\exp\left(\frac{-\Delta\eta^2}{4\delta^2}\right)$$

Effective cluster size

Cluster width

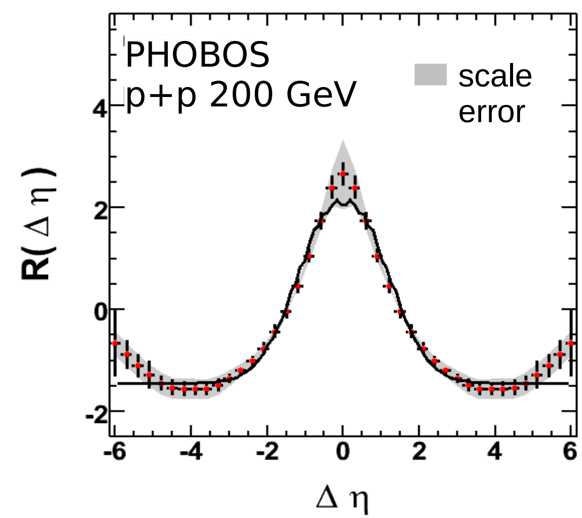
Possible source for correlations:
Production of intermediate objects which decay into particles

p+p@200GeV




Project onto Δη axis and fit with a simple parametrization

NB: In A+A this removes elliptic flow



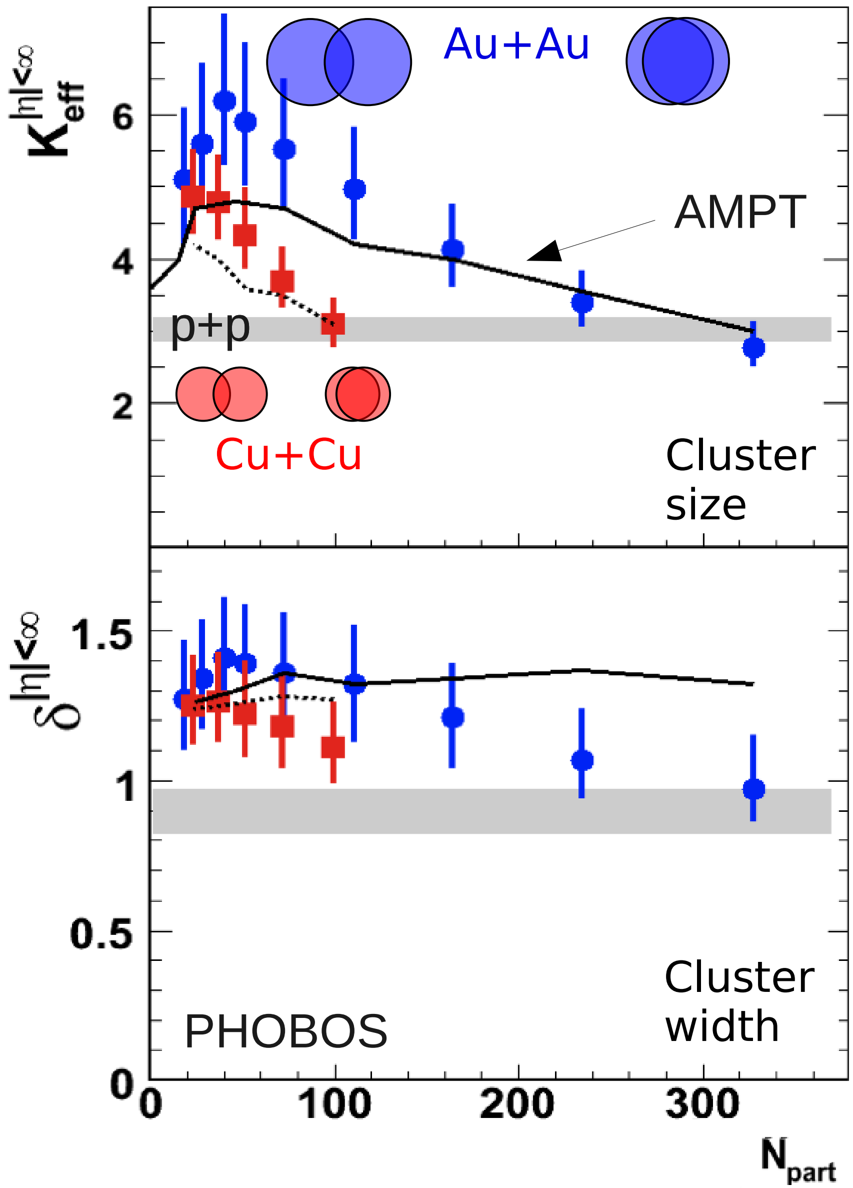

PHOBOS, PRC 75 054913 (2007)
PHOBOS, PRC 81 024904 (2010)

# Cluster model fit results



Multiplicity of the clusters is large
(up to 6 charged particles - more than for known resonances)

Cluster width exceeds that for isotropic decay at rest (~1)




NB: Extrapolated to full phase space

PHOBOS, PRC 75 054913 (2007)
PHOBOS, PRC 81 024904 (2010)

# Cluster model fit results

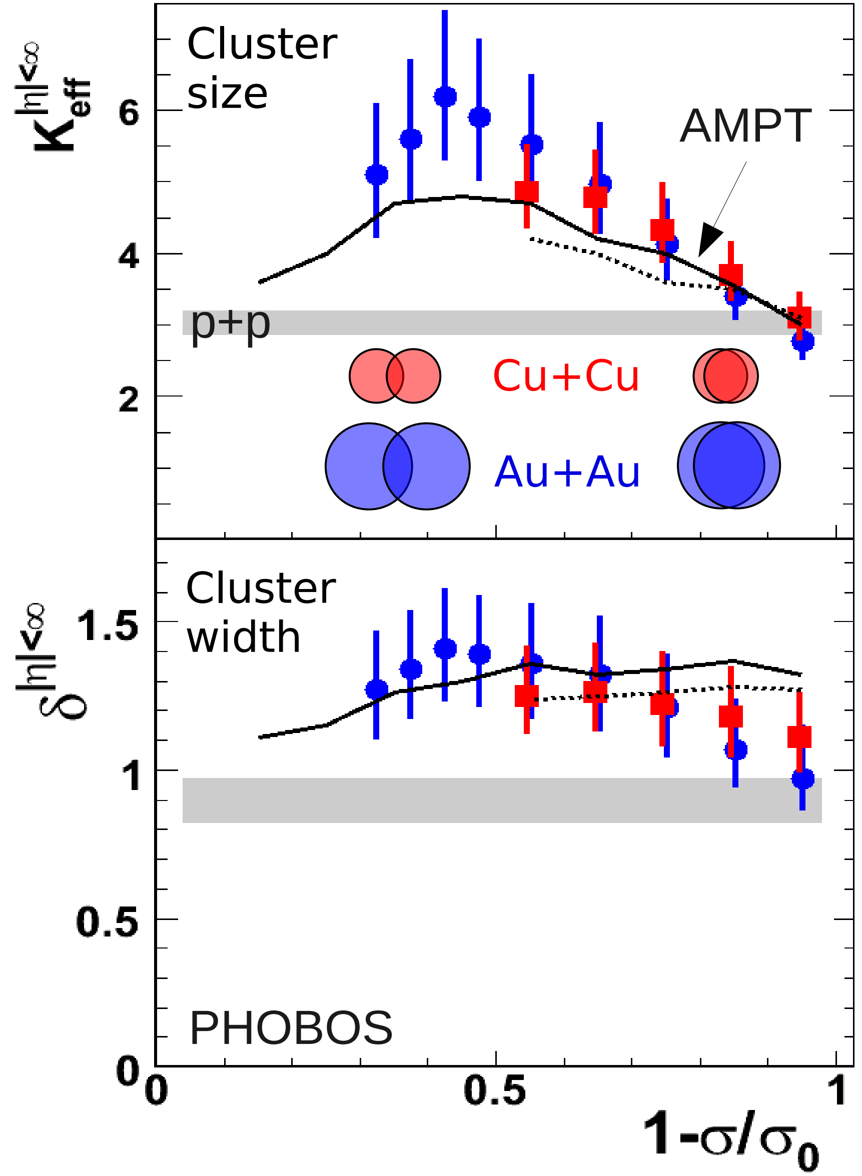


Multiplicity of the clusters is large (up to 6 charged particles - more than for known resonances)

Cluster width exceeds that for isotropic decay at rest (~1)

Cluster sizes and width very similar at the same centrality, defined as the same fraction of cross section.

The shape of the interaction area seems to even determine the properties at hadronization?



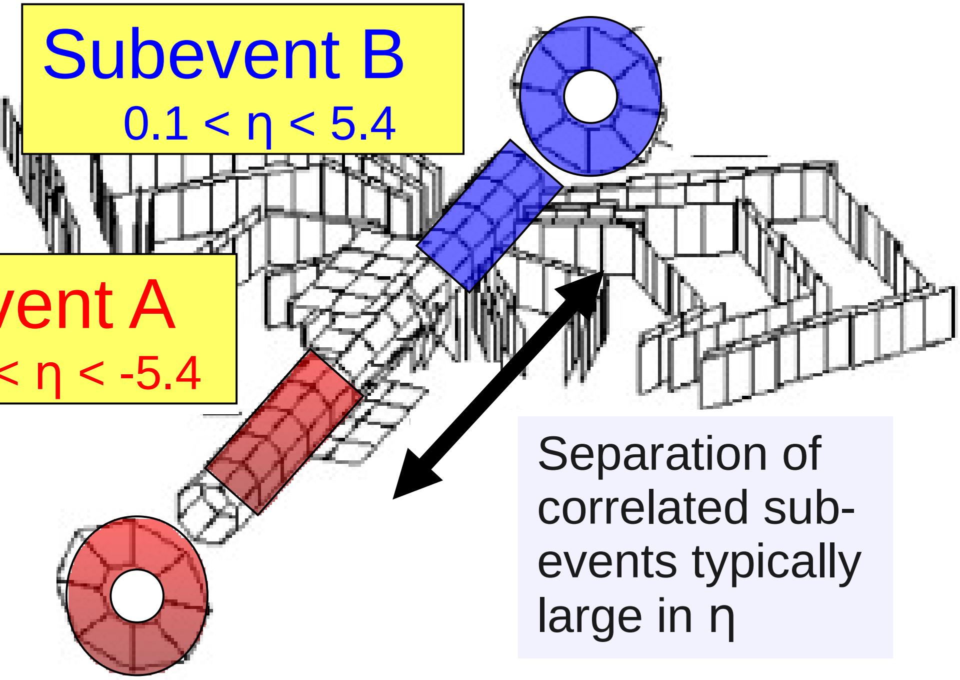

NB: Extrapolated to full phase space

PHOBOS, PRC 75 054913 (2007)
PHOBOS, PRC 81 024904 (2010)

# Elliptic flow measurement in PHOBOS



- Reaction-plane / Sub-event technique
  - Correlate reaction plane determined from azimuthal pattern of hits in one part of the detector with information from other parts a of the detector




$$\tan(2\psi_A) = \frac{\langle \sin(2\phi)\rangle_A}{\langle \cos(2\phi)\rangle_A}$$

$$v_2^{obs} = \langle \cos(2\phi - 2\psi_A)\rangle_B$$

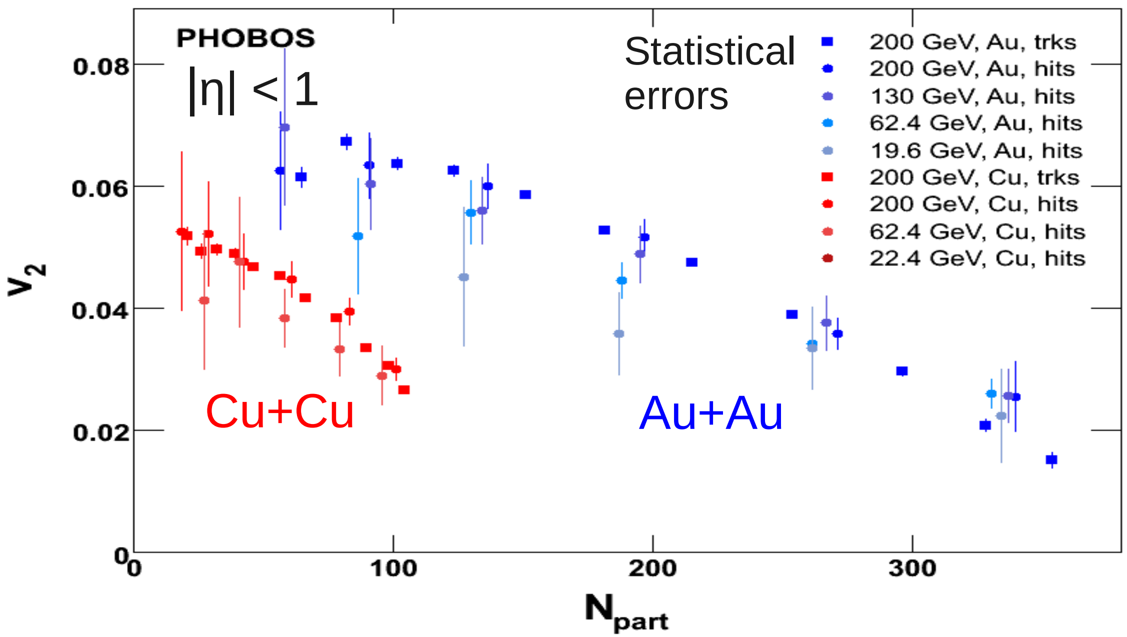
$$v_2 = \frac{\langle v_2^{obs}\rangle_{events}}{\sqrt{\langle \cos(2\psi_A - 2\psi_B)\rangle_{events}}}$$

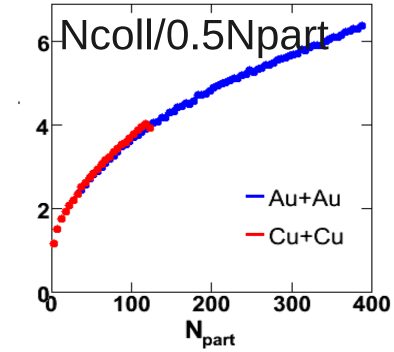
Poskanzer, Voloshin, nucl-ex/9805001

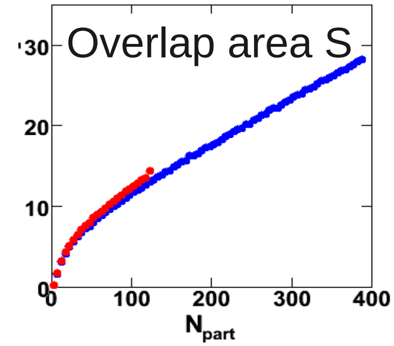
Resolution correction

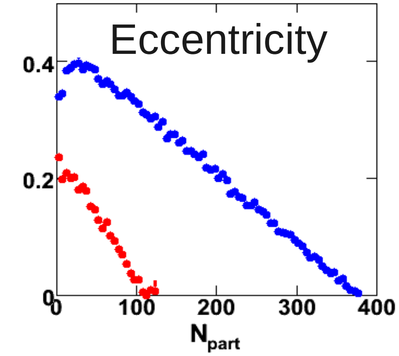
# Elliptic flow and collision geometry





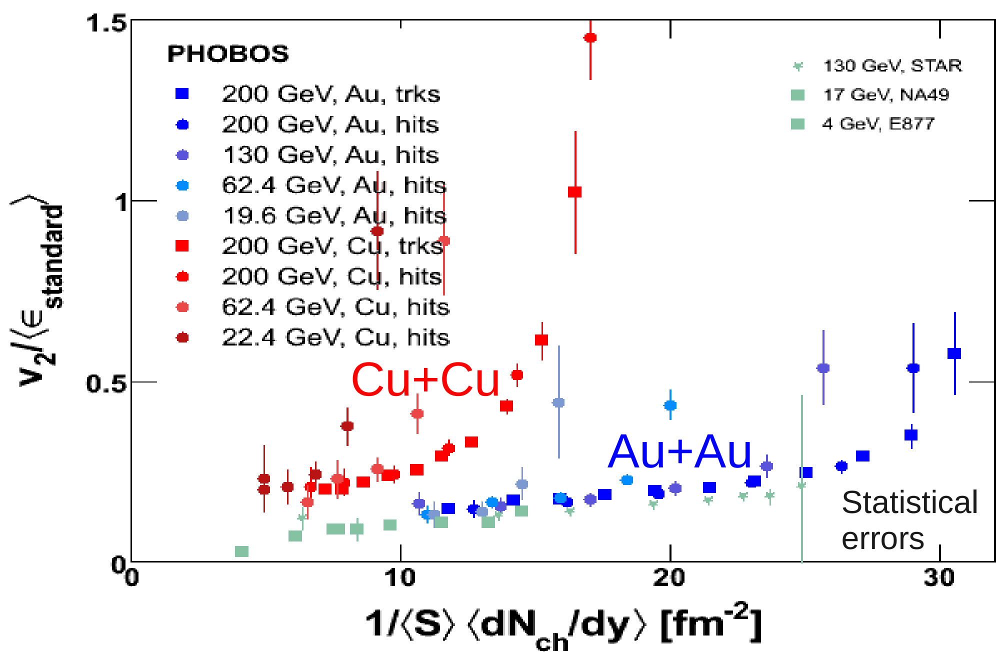



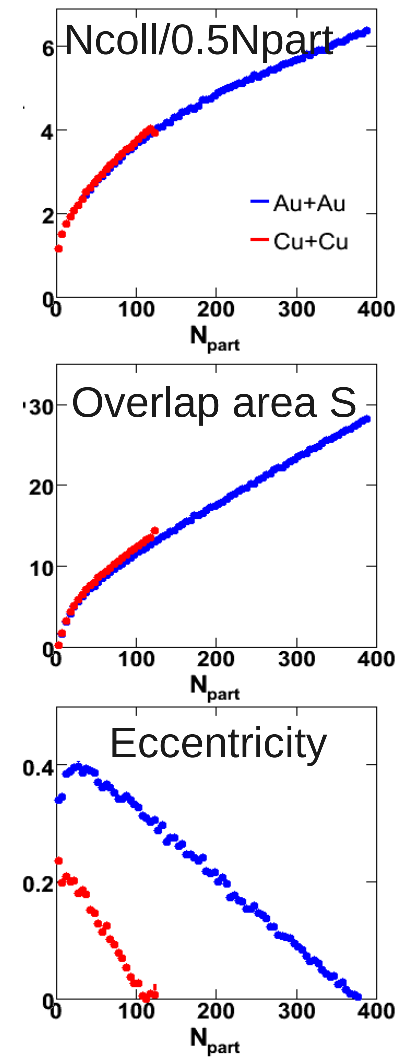

Geometry should cancel out in the $v_2/\varepsilon$ ratio

PHOBOS, Au+Au, 200,130,62.4+19.6 GeV: PRL 94 122303 (2005)
PHOBOS, Cu+Cu, 200+62.4 GeV: PRL 98 242302 (2007)
PHOBOS, Cu+Cu, 22.4 GeV: prel. QM06

# Elliptic flow and collision geometry





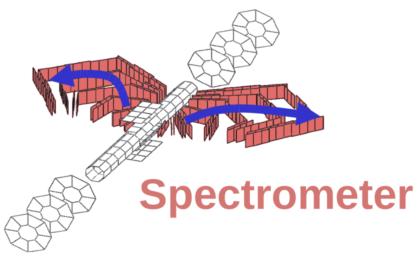



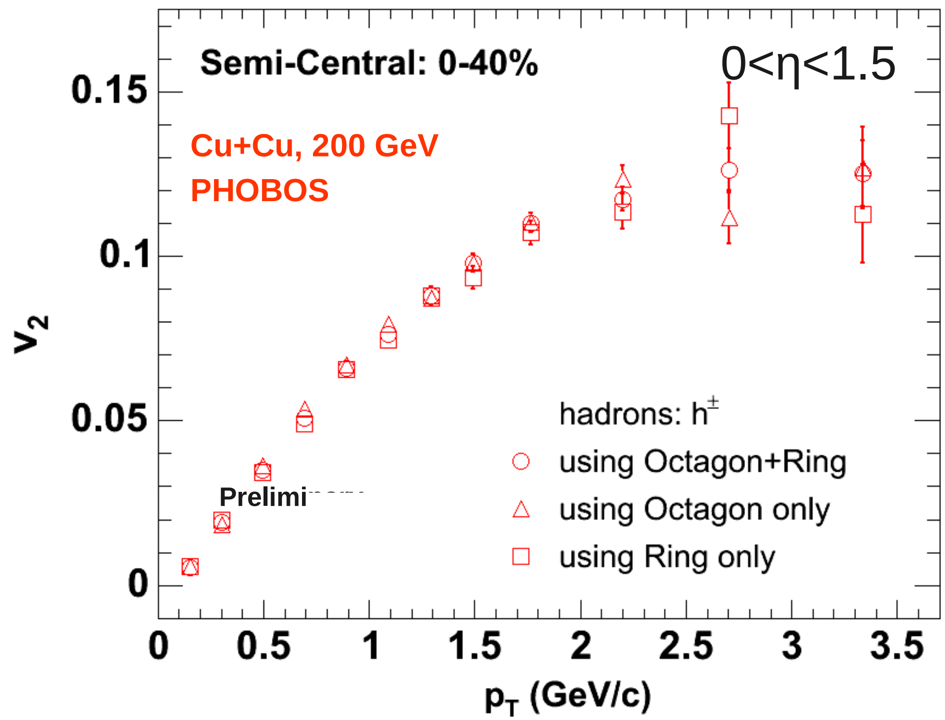

No scaling between Cu+Cu and Au+Au
using the standard eccentricity definition

PHOBOS, Au+Au, 200,130,62.4+19.6 GeV: PRL 94 122303 (2005)
PHOBOS ,Cu+Cu, 200+62.4 GeV: PRL 98 242302 (2007)
PHOBOS, Cu+Cu, 22.4 GeV: prel. QM06
STAR+NA49+E877, PRC 66 034904 (2002)
(data taken with no adjustments)

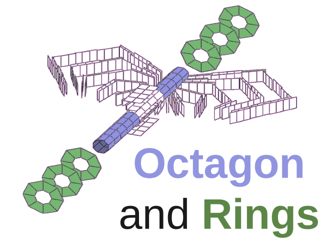
# Study non-flow contribution via eta gaps 

**Track-based method**

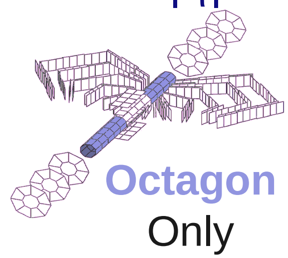



$2.0 < |\eta| < 5.4$



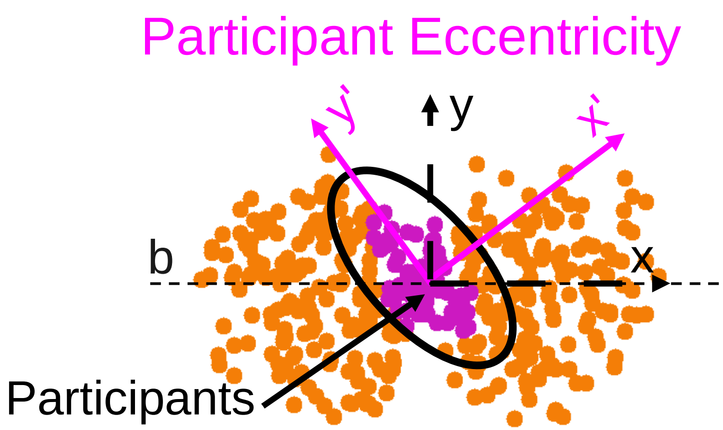

$2.0 < |\eta| < 3.2$

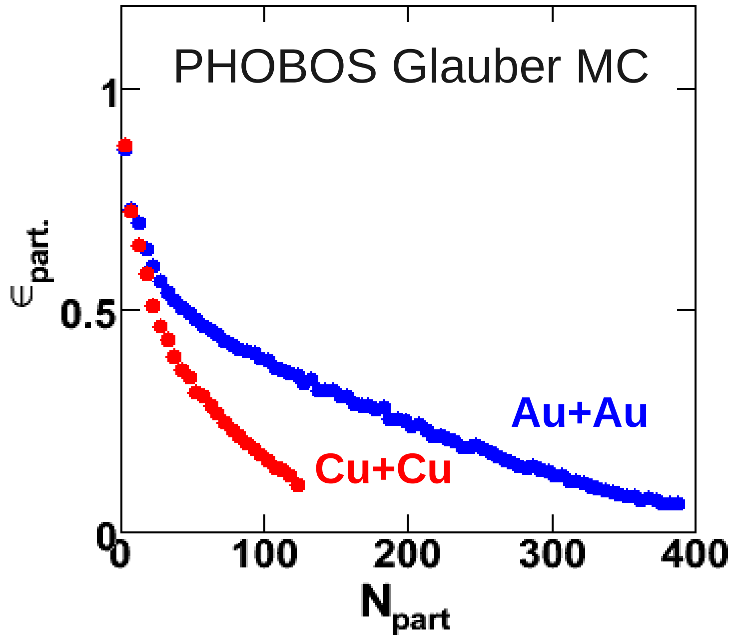



$3.0 < |\eta| < 5.4$


The large η separation between the subevents and the signal region suppresses the non-flow contribution in track-based (hit-based) method.

# Participant eccentricity




The spatial distribution of the interaction points of participating nucleons for the same b varies from event-to-event. Thus, define

$$\epsilon_{part} = \frac{\sqrt{(\sigma_y^2 - \sigma_x^2)^2 + 4\sigma_{xy}^2}}{\sigma_y^2 + \sigma_x^2}$$

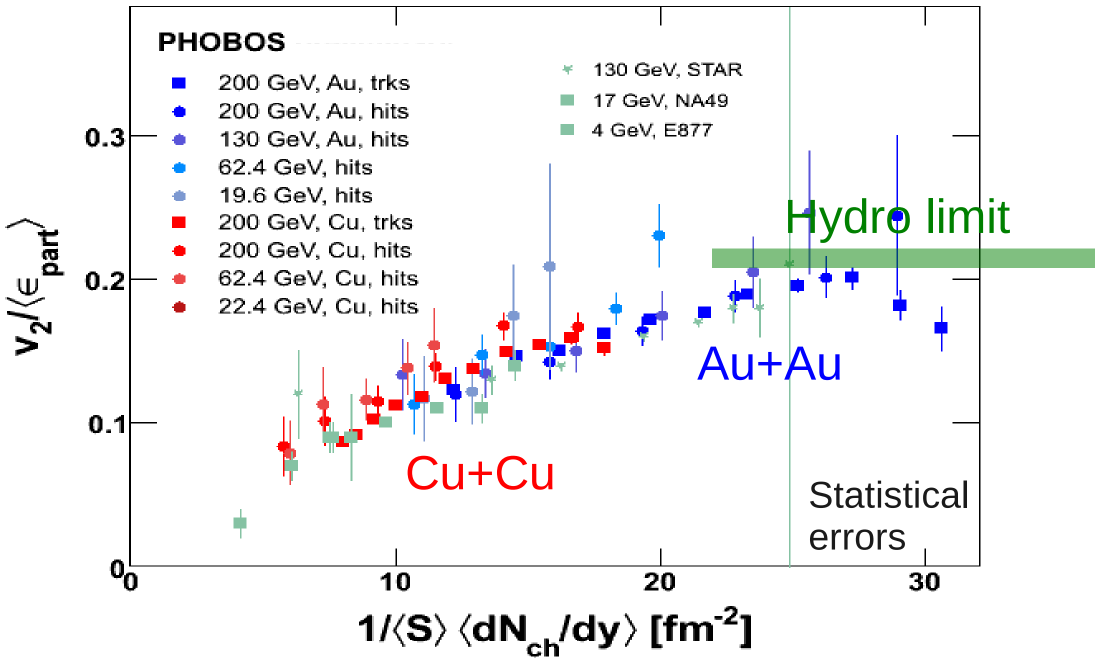
Co-variance term not in standard definition

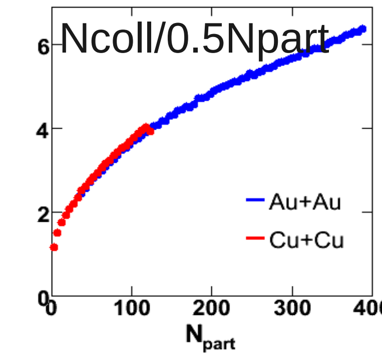
$$(0 < \epsilon_{part} \leq 1)$$

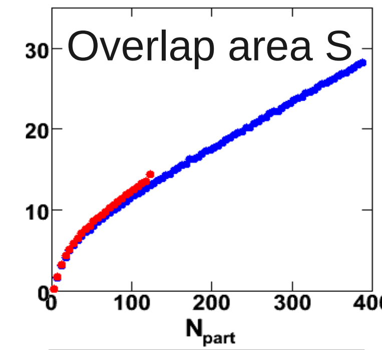
Introduced at QM05,
PHOBOS, PRL 98 242302 (2007)

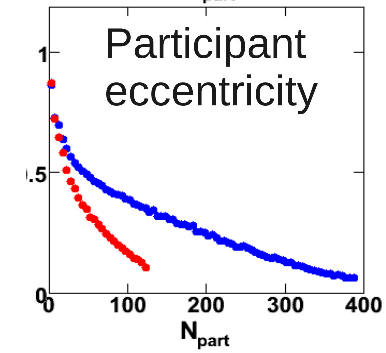
# Elliptic flow and collision geometry





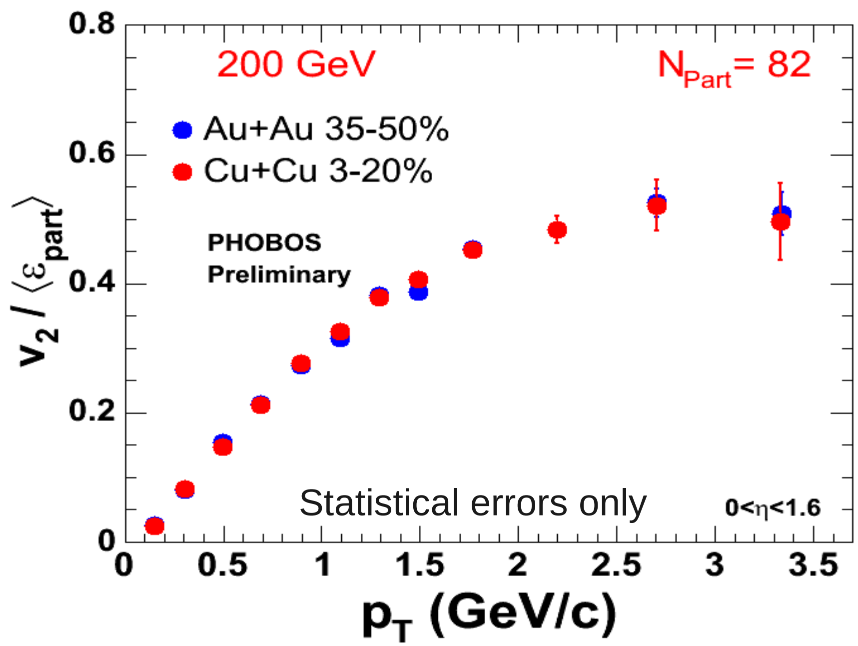



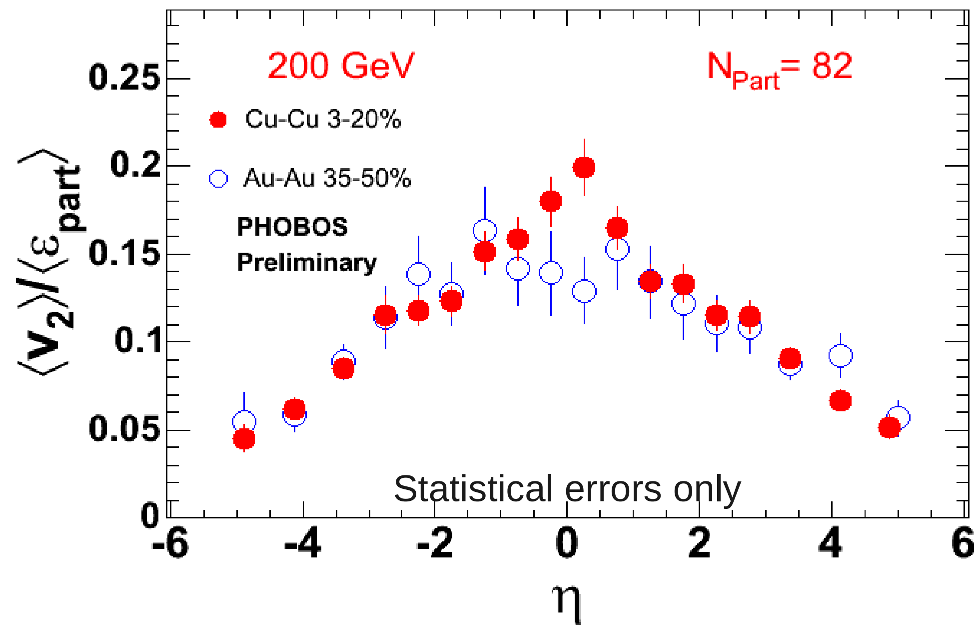

Scaling between Cu+Cu and Au+Au
using participant eccentricity definition

PHOBOS, Au+Au, 200,130,62.4+19.6 GeV: PRL 94 122303 (2005)
PHOBOS ,Cu+Cu, 200+62.4 GeV: PRL 98 242302 (2007)
PHOBOS, Cu+Cu, 22.4 GeV: prel. QM06
STAR+NA49+E877, PRC 66 034904 (2002)
(data taken with no adjustments)

# Eccentricity scaling is global





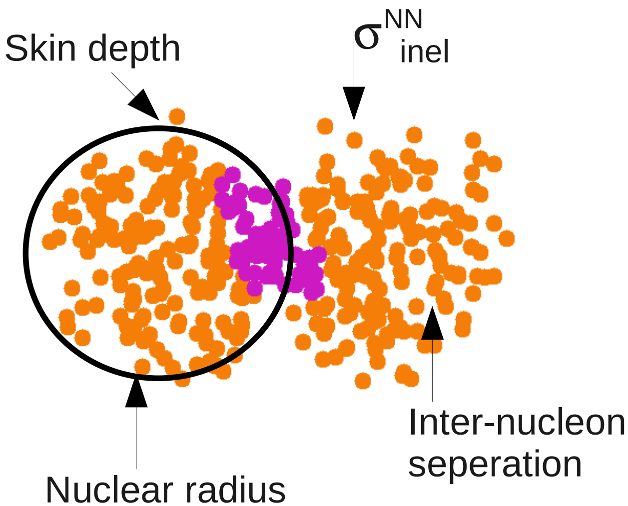



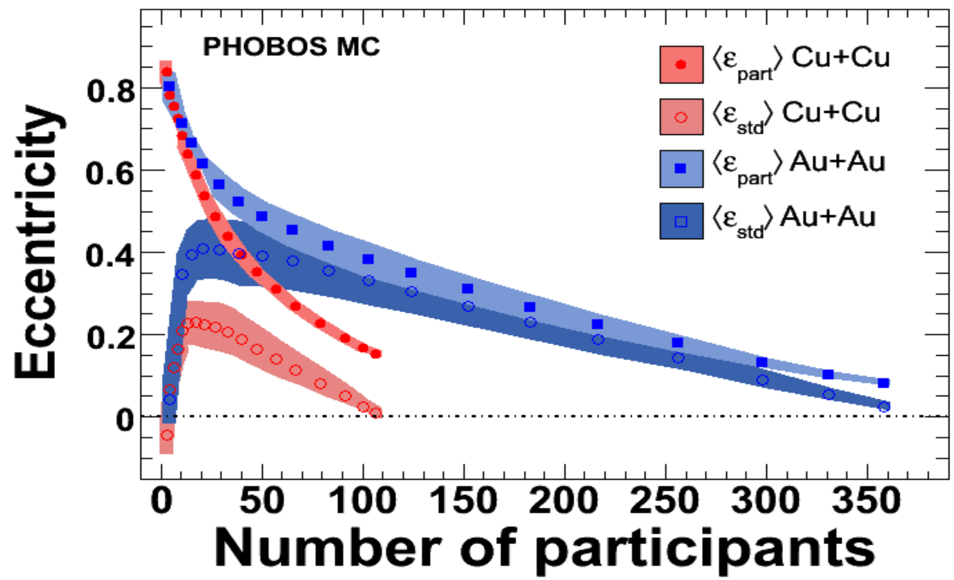

Unity of geometry, system, energy, transverse momentum and pseudorapidity for the same $N_{part}$ (~area density)

PHOBOS, JPG 34 887 (2007)

# Robustness of eccentricity definition






Baseline parameters:

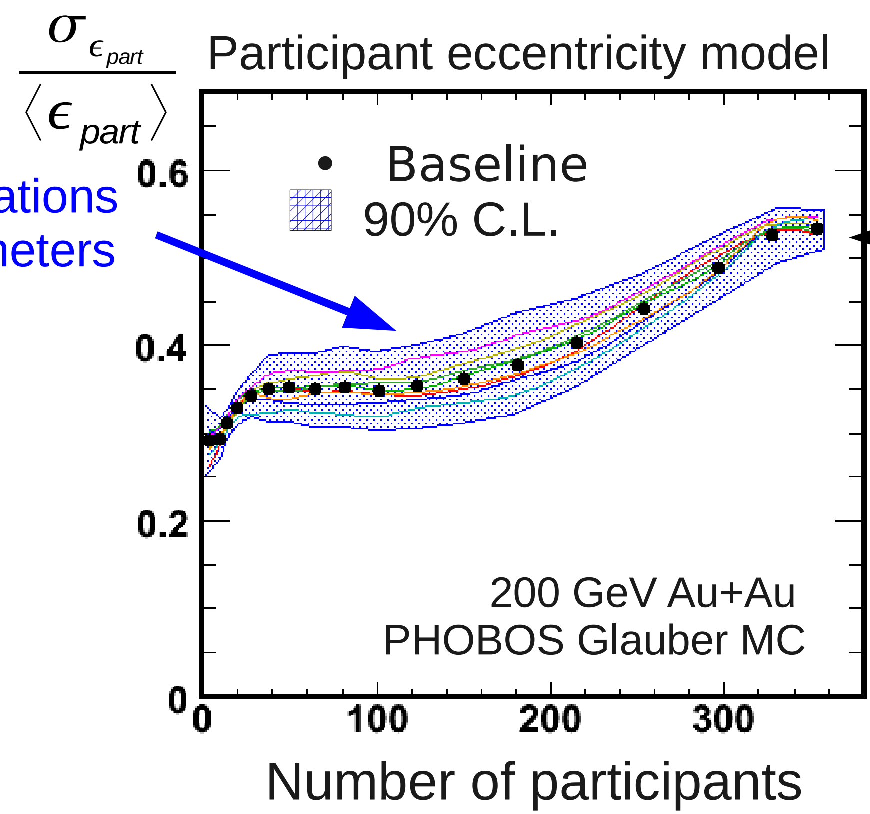
- Nucleon-nucleon cross section: $\sigma_{NN}$=42mb
- Skin depth: a=0.535fm
- Wood-saxon radius: $R_A$=6.38fm
- Inter-nucleon separation distance: d=0.4fm

Robust definition wrt variation of Glauber parameters and to varying assumptions about matter production
(the latter not shown here, see extra slides)

PHOBOS+Heinz, PRC 77 014906 (2008)

# Expected relative flow fluctuations 

Uncertainty from variations of Glauber MC parameters




Analytic (b=0fm)

$$\sqrt{\frac{4}{\pi}-1} \approx 0.52$$

Broniowski et al., PRC 76 054905 (2007)

Baseline parameters:

- Nucleon-nucleon cross section: $\sigma_{NN}$=42mb
- Skin depth: a=0.535fm
- Wood-saxon radius: $R_A$=6.38fm
- Inter-nucleon separation distance: d=0.4fm

If initial state fluctuations are present, expect large relative flow fluctuations:

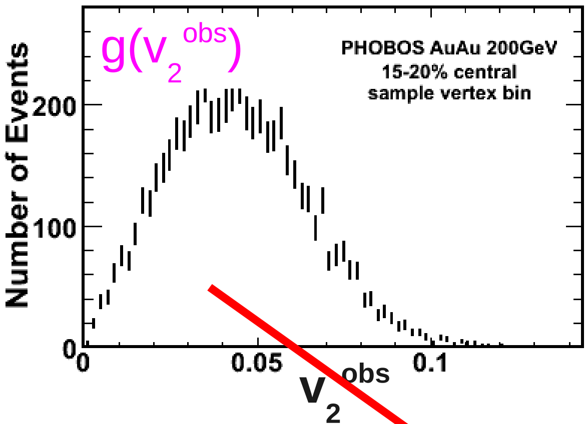
$$\frac{\sigma_{v_2}}{\langle v_2 \rangle} \sim \frac{\sigma_{\epsilon_{part}}}{\langle \epsilon_{part} \rangle}$$

PHOBOS, nucl-ex/0608025

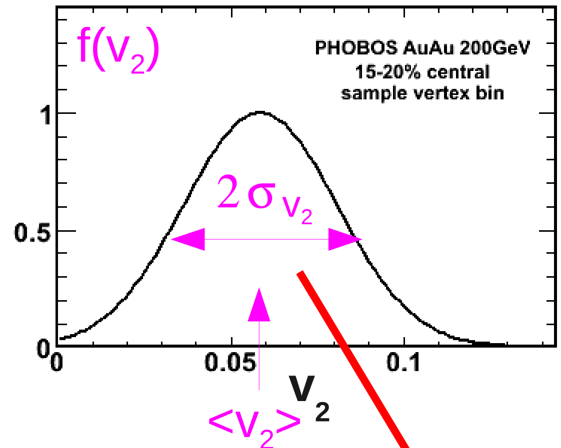
# Measuring elliptic flow fluctuations



Observed $v_2$ distribution




Parametrized $v_2$ distribution



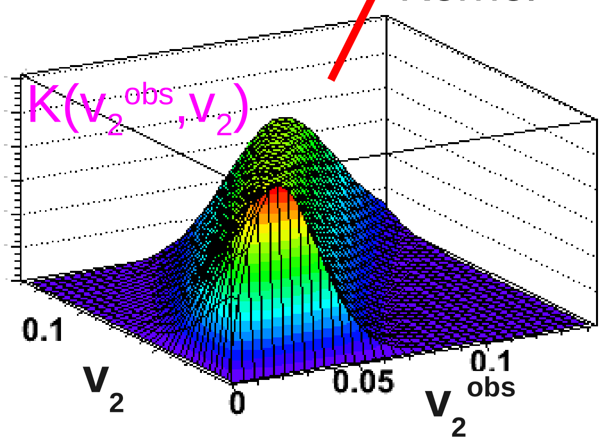

Max-Likelihood fit to determine: $\langle v_2 \rangle$ and $\sigma_{v_2}$

Kernel

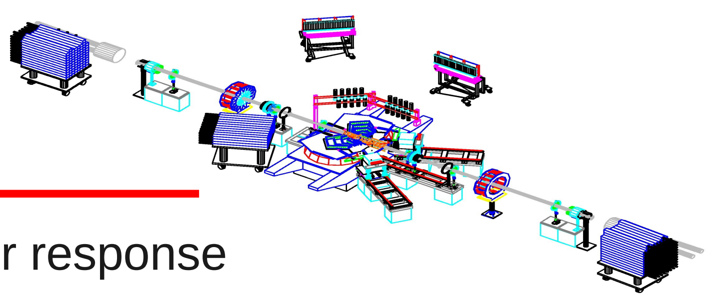
- Detector and acceptance effects
- Finite-number fluctuations
- Multiplicity fluctuations

$$g(v_2^{obs}) = \int_0^1 K(v_2^{obs}, v_2)\, f(v_2)\, dv_2$$

Kernel

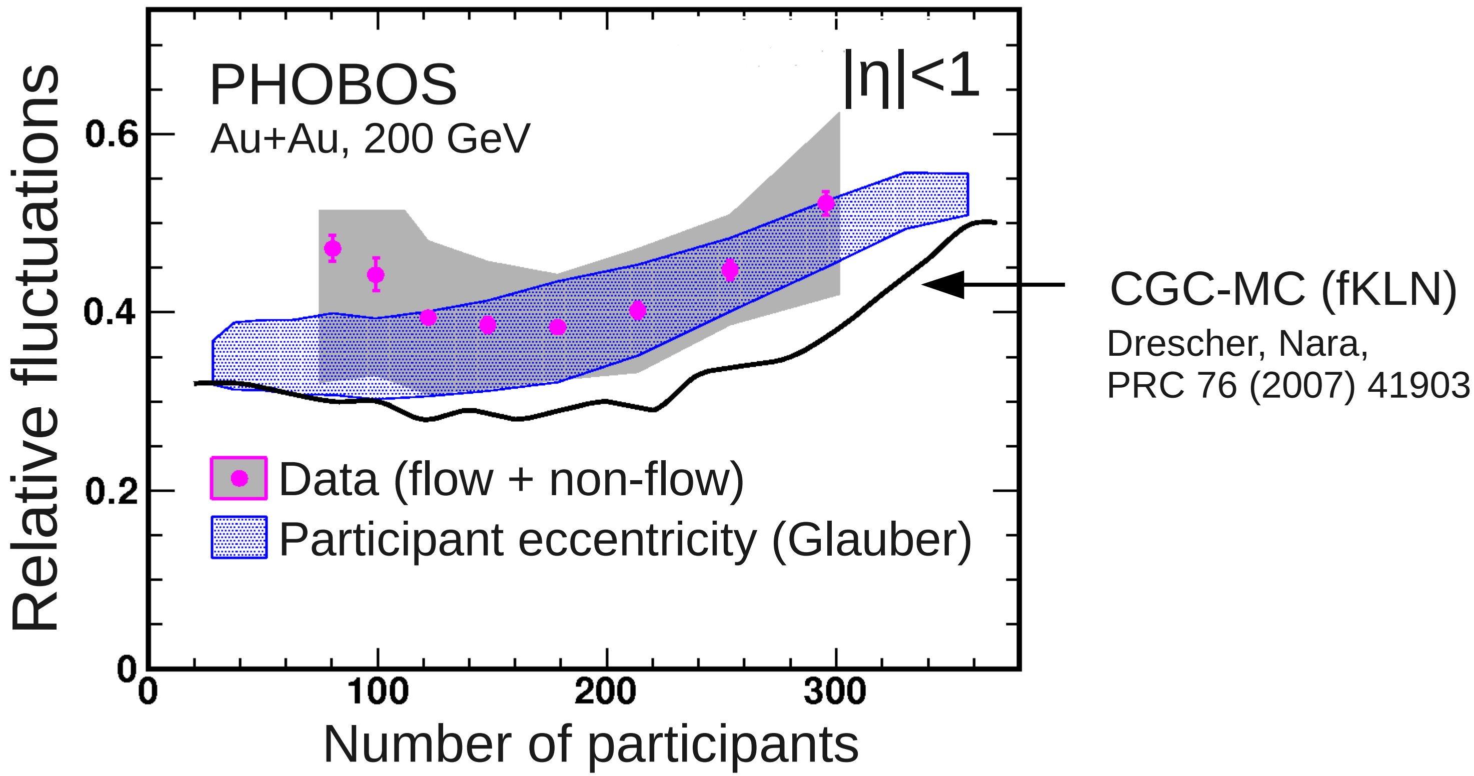



Detector response



PHOBOS, PRL 104 142301 (2010)

# Measured relative total fluctuations




Shown at QM06 as flow fluctuations, however non-flow contribution (included in sys.error) was underestimated.

Now interprete as total $v_2$ fluctuations.

PHOBOS, PRL 104 142301 (2010)

# Measure non-flow contribution



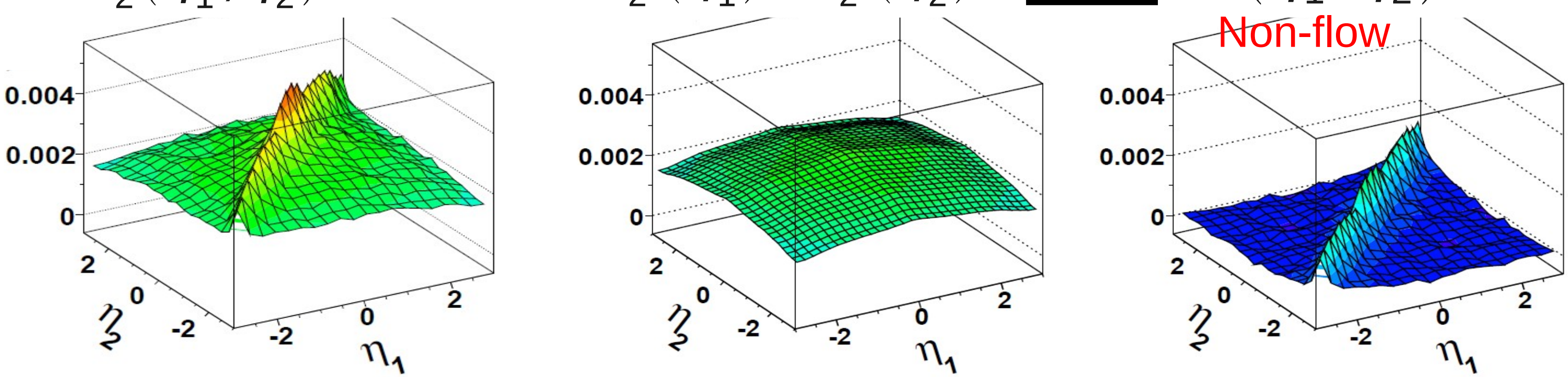
PHOBOS uses data driven analysis to measure the contribution of non-flow

- Flow is a function of η and correlates particles at all Δη
- Non-flow (δ) is dominated by short range correlations (small Δη)
- Study correlations at different Δη

$$v_2^2(\eta_1,\eta_2) \equiv \langle \cos(2\Delta\phi)\rangle(\eta_1,\eta_2)$$
$$= v_2(\eta_1) * v_2(\eta_2) + \delta(\eta_1,\eta_2)$$

- Assume non-flow to be zero for Δη>2
- Fit $v_2^2(\eta_1,\eta_2) = v_2^{fit}(\eta_1) * v_2^{fit}(\eta_2)$ , $|\eta_2 - \eta_1| > 2$
- Subtract fit results at all $(\eta_1,\eta_2)$
- Integrate over particle pairs to obtain $\delta / v_2^2$
- Numerically relate $\delta / v_2^2$ and $\sigma_{v_2} / \langle v_2 \rangle$ to obtain $\sigma_{flow} / \langle v_2 \rangle$

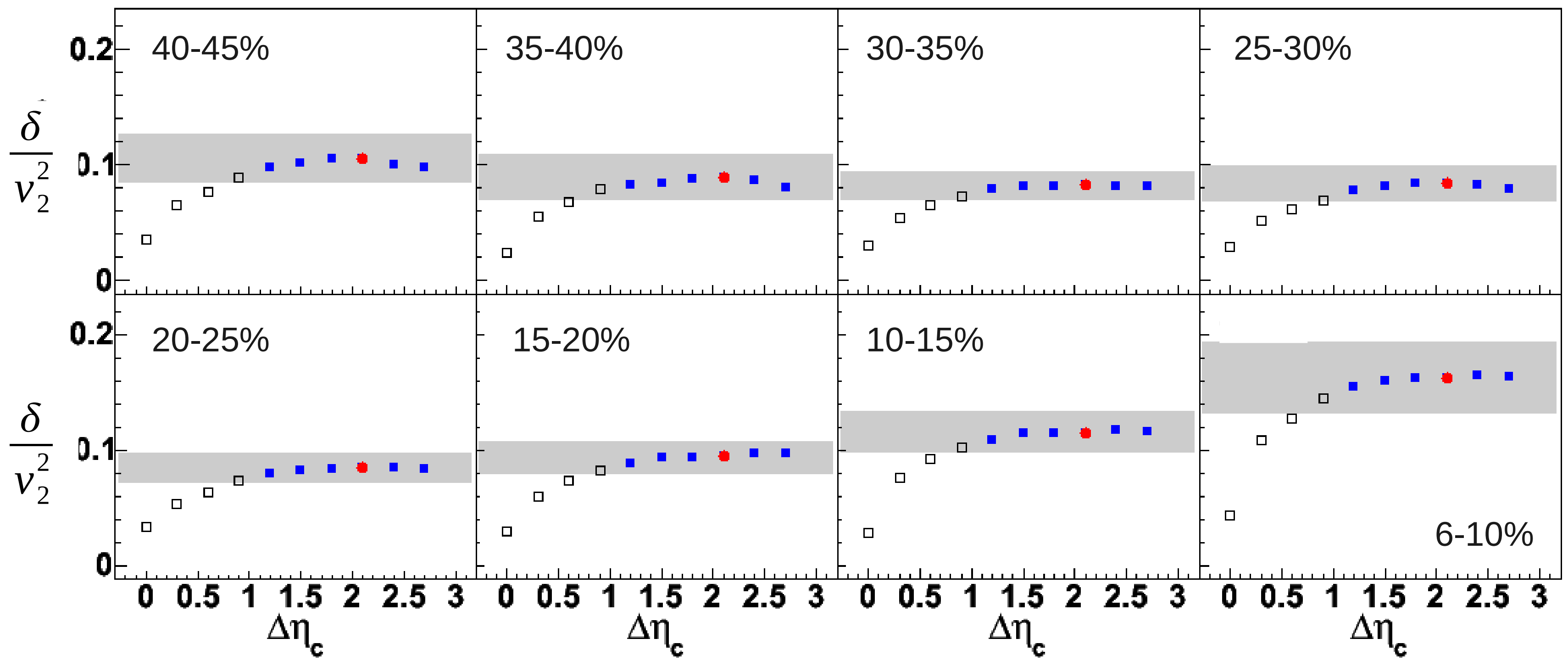
$$v_2^2(\eta_1,\eta_2) - v_2^{fit}(\eta_1) \times v_2^{fit}(\eta_2) = \delta(\eta_1,\eta_2)$$




PHOBOS, PRC 81 034915 (2010)

# Variation of the fit region






Non-flow ratio as a function of Δη cut used to define the fit region.

Saturation is very encouraging, however does not rule out contributions with very little Δη dependence.

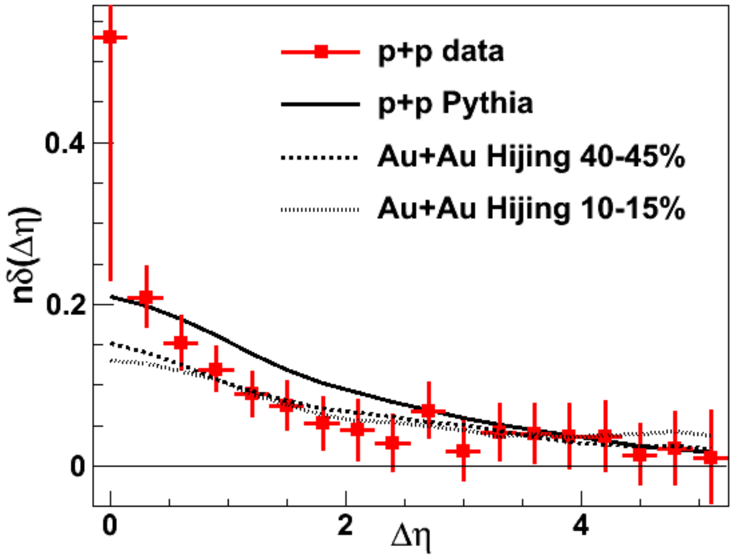
Red-point is baseline for analysis, while black points are used for systematic error

PHOBOS, PRC 81 034915 (2010)

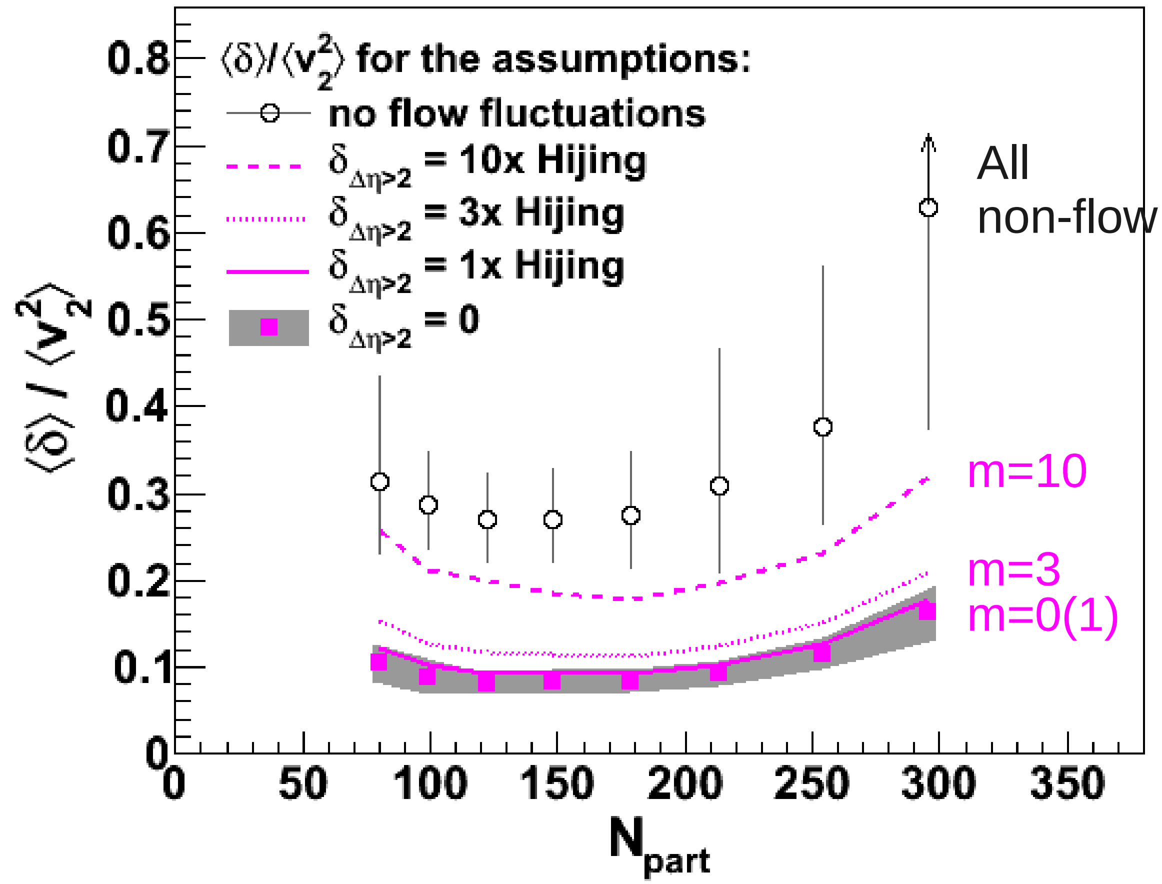
# Variation of non-flow strength in fit region



Non-flow in p+p, 200 GeV




$\delta / v_2^2$ for assumptions in Au+Au, 200 GeV




Measure “non-flow” in p+p data, and compare to MC generators

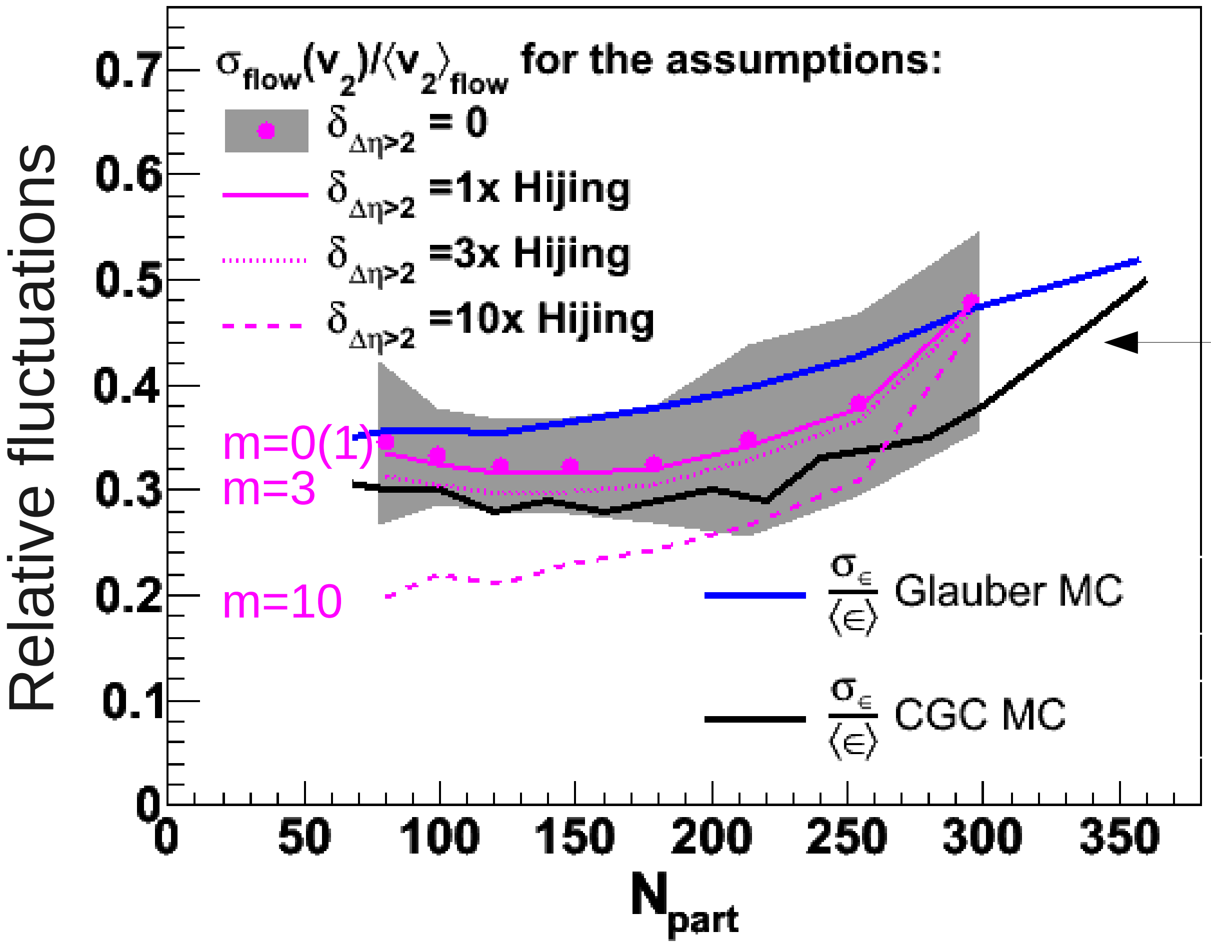
Assume non-flow in fit region to be
m times non-flow in p+p (rather than 0)

$$v_2^2(\eta_1,\eta_2) - m\,\delta_{MC}^{HIJING} = v_2^{fit}(\eta_1) * v_2^{fit}(\eta_2) \quad |\eta_2 - \eta_1| > 2$$

PHOBOS, PRC 81 034915 (2010)

# Measured relative flow fluctuations

For m=3
80-95% of
$\sigma_{v_2}/\langle v_2 \rangle$




CGC-MC (fKLN)

Drescher, Nara,
PRC 76 (2007) 41903

Initial state fluctuations if indeed present seem not to be significantly enhanced in later stages of the collision.

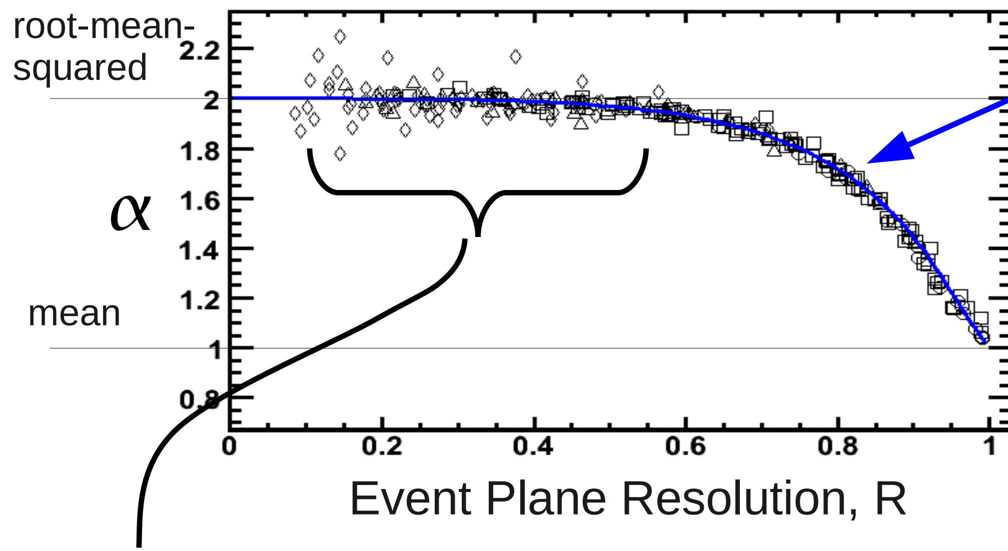
Short-range (Δη<2) non-flow contribution are removed

PHOBOS, PRC 81 034915 (2010)

# Which moment of $v_2$ is measured?

Define

$$v_2 \equiv \langle v_2^{\alpha} \rangle^{1/\alpha}$$




By now α is known:

$$\alpha = 2 - 4 i_1^2/(i_0 + i_1)^2$$

Ollitrault et. al., PRC 80 80 014904 (2009)

PHOBOS R: 0.13 – 0.55

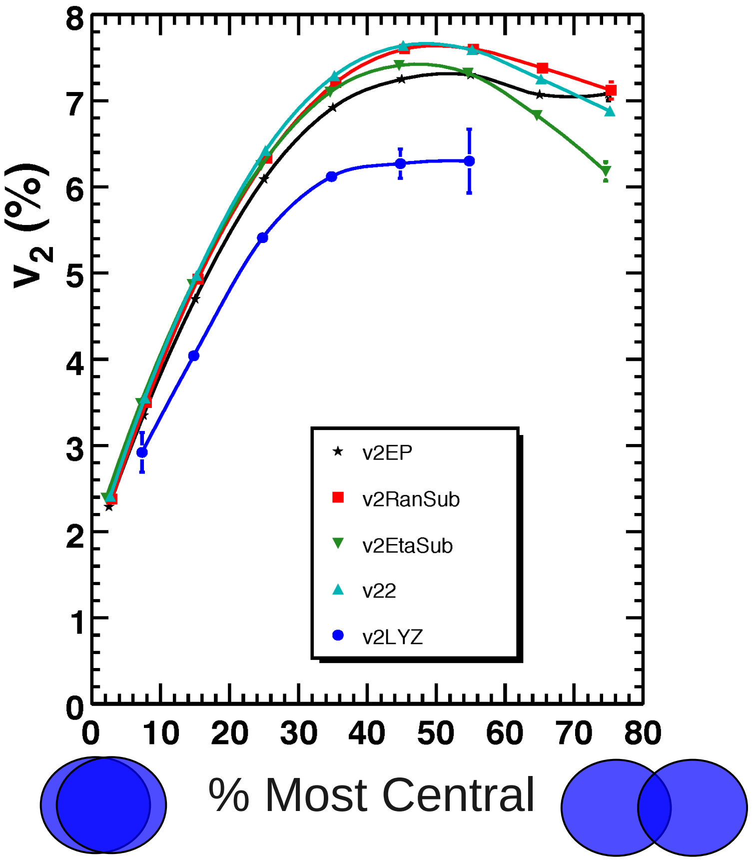
For PHOBOS standard event-plane method $v_2\{EP\} = \sqrt{\langle v_2^2 \rangle}$

(For the observed fluctuations this implies up to about 10% difference)

PHOBOS+Heinz, PRC 77 014906 (2008)

# Correction for non-flow and fluctuations 

Published STAR results




Derive analytic correction for non-flow and fluctuations in leading order of $\delta$ and $\sigma^2_{v_2}$

$$v_2\{2\}^2 = \langle v_2 \rangle^2 + \sigma^2_{v_2} + \delta$$

$$v\{4\}^2 = \langle v \rangle^2 - \sigma^2_{v_2}$$

$$v\{\mathrm{subEP}\}^2 = \langle v \rangle^2 + (1 - f(R))\,\sigma^2_{v_2} + (1 - 2f(R))\,\delta$$

$$0 \le f(R) < 0.2$$

Differences between methods proportional to

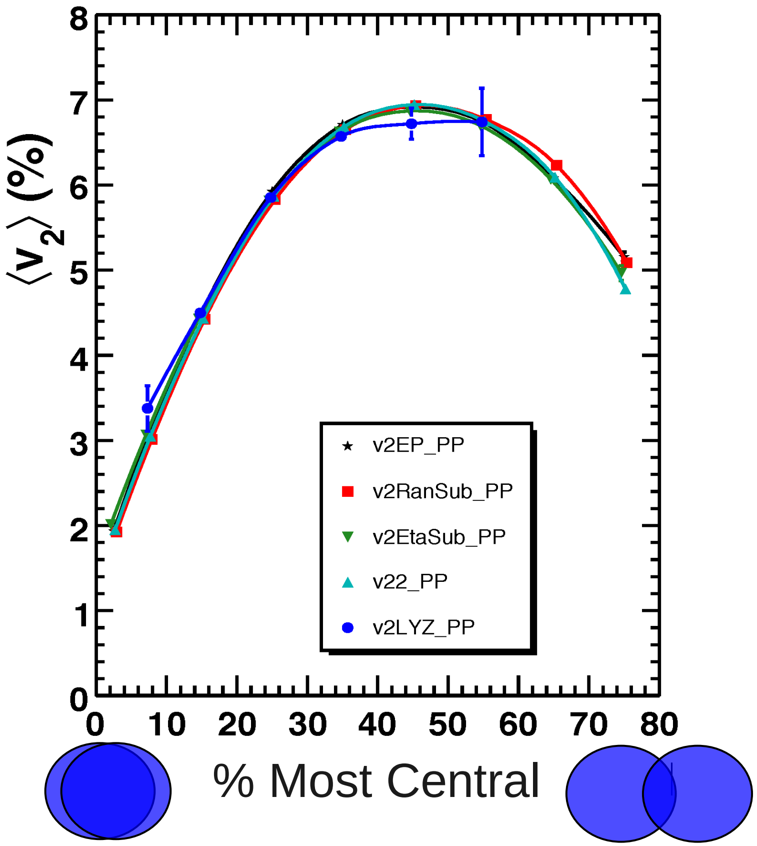
$$\sigma_{tot} = \delta + 2\,\sigma^2_{v_2}$$

Need additional assumption or information to separate between non-flow and fluctuations

Ollitrault, Poskanzer, Voloshin
PRC 80 80 014904 (2009)

# Correction for non-flow and fluctuations

Corrected mean results



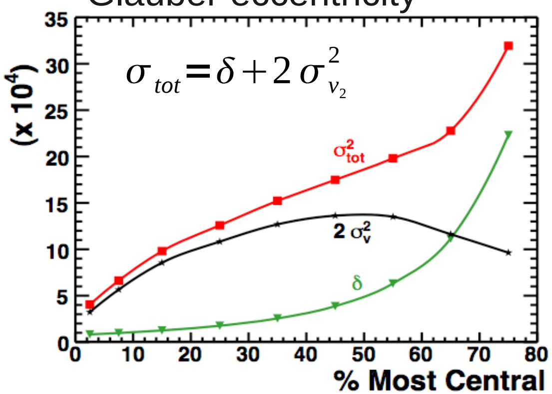

Model assuming:

$$\sigma_{v_2} = \frac{\sigma_{\epsilon_{\mathrm{part}}}}{\langle \epsilon_{\mathrm{part}} \rangle} \langle v_2 \rangle$$

$$\delta = \frac{2}{N_{\mathrm{part}}} \delta_{\mathrm{pp}}$$

with $\delta_{\mathrm{pp}} = 0.0145$

Glauber eccentricity



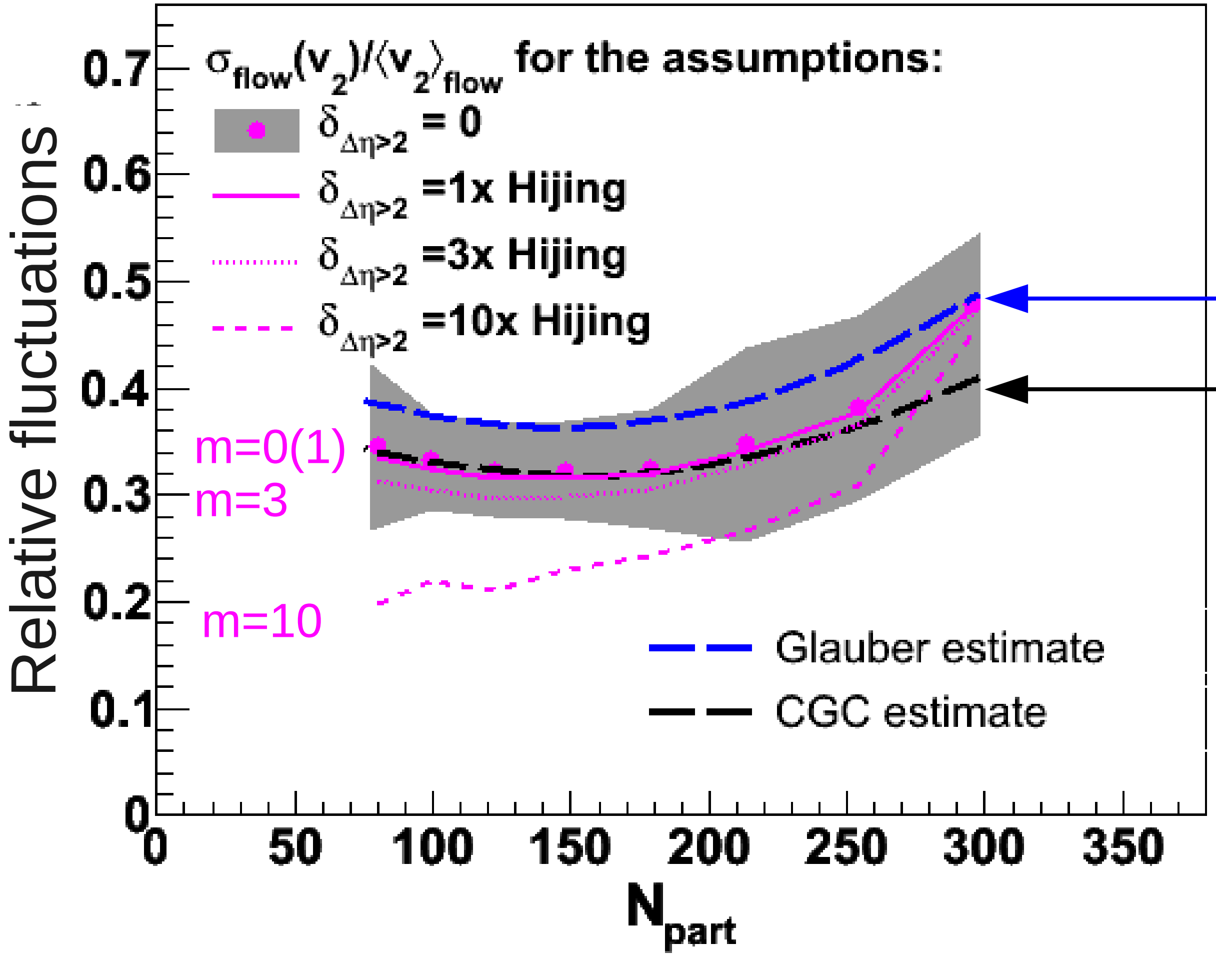

Corrected mean values agree in participant frame.
Reduces errors on $v_2$ measurements by about 20%.

Eccentricity values are calculated for standard Glauber and a mix of 30:70 CGC (not shown)

Ollitrault, Poskanzer, Voloshin
PRC 80 80 014904 (2009)

# Measured relative flow fluctuations






Model corrections:

Glauber

CGC (30:70)

Ollitrault et. al.,
PRC 80 80 014904 (2009)

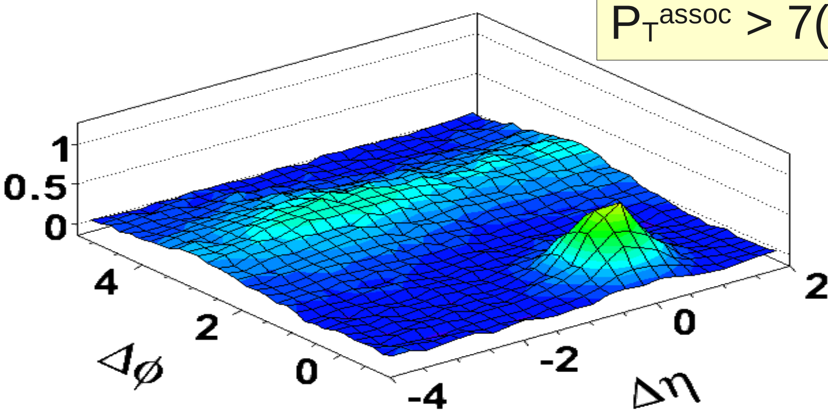
Results based on analytic model tuned to STAR data applied to total measured fluctuations are consistent with PHOBOS data.

# Correlations wrt trigger particle 

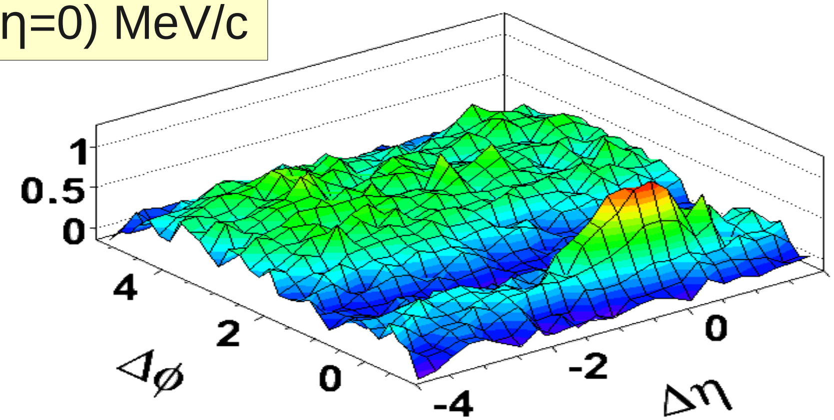
$$\frac{1}{N_{trig}} \frac{d^2 N_{ch}}{d\Delta\phi\, d\Delta\eta}$$

$p_T^{trig}$ >2.5 GeV/c, 0<$\eta^{trig}$<1.5
$P_T^{assoc}$ > 7(η=3), >35(η=0) MeV/c

p+p
(PYTHIA)




Au+Au 0-30%
(PHOBOS)




$$\frac{1}{N_{trig}} \frac{d^2 N_{ch}}{d\Delta\phi\, d\Delta\eta} = B(\Delta\eta)\left\{ \boxed{\frac{s(\Delta\phi,\Delta\eta)}{b(\Delta\phi,\Delta\eta)}} - a(\Delta\eta)\left[1 + 2V(\Delta\eta)\cos(2\Delta\phi)\right]\right\}$$

Normalization

Raw correlation

Scale factor

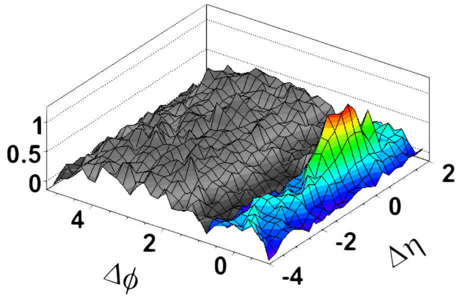
Elliptic flow

NB: PYTHIA agrees with STAR at mid-rapidity for a similar set of $p_T$ cuts

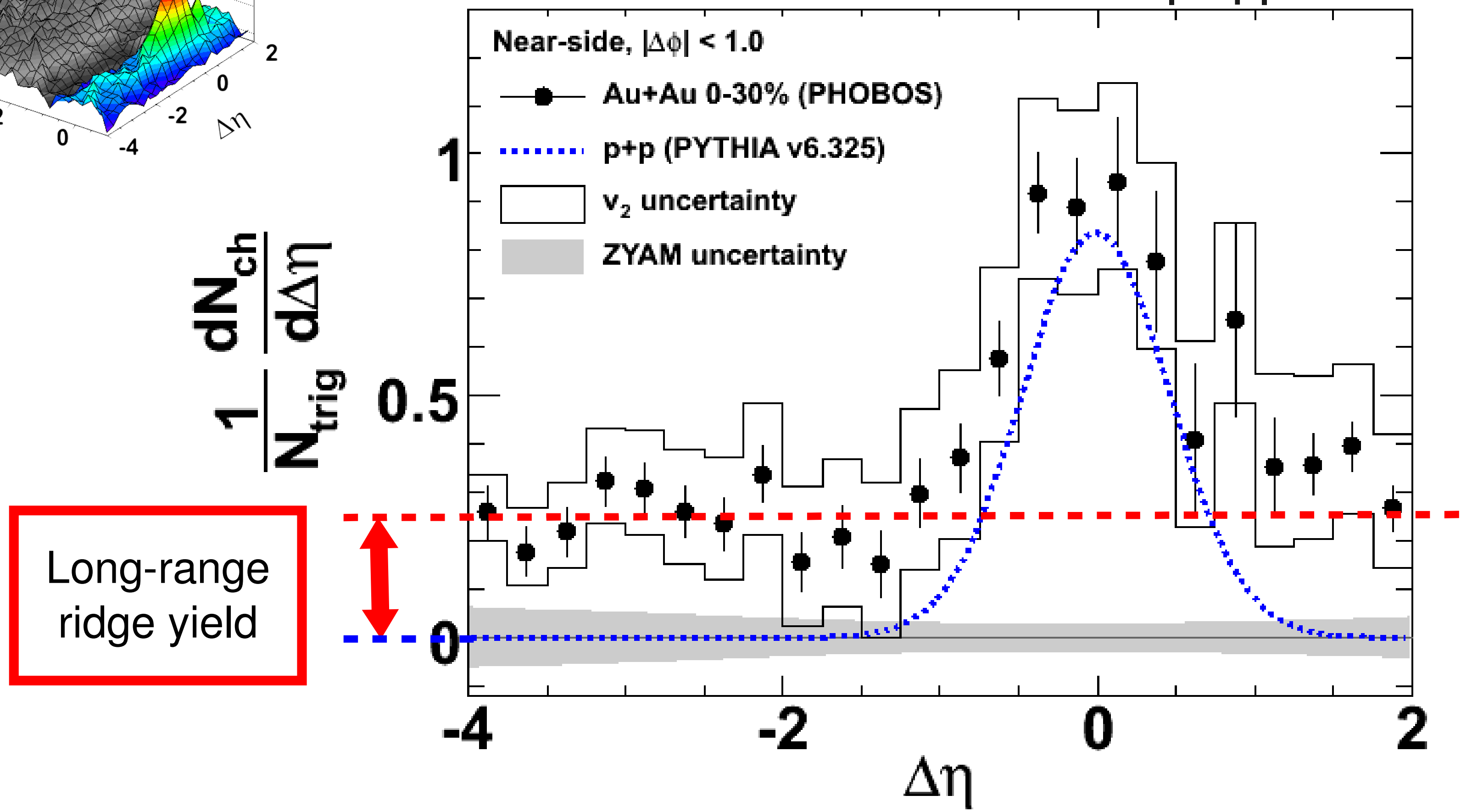
PHOBOS, PRL 104 062301 (2010)

# Ridge extent in Δη (near-side)





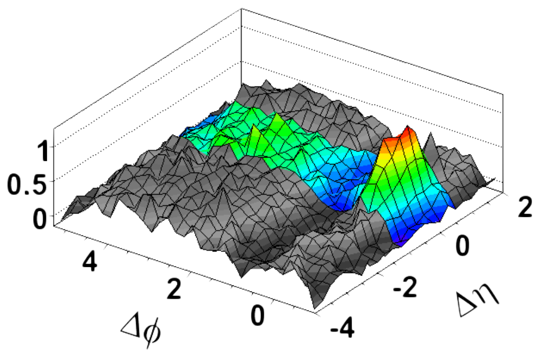

Near-side correlation for |Δφ|<1



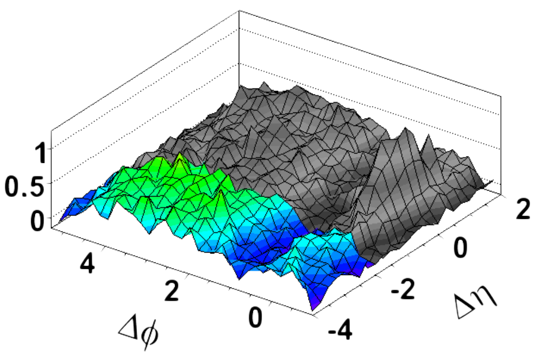

In 0-30% most central Au+Au, there is a relatively flat correlated yield of about 0.25 particles per trigger particle (per unit η)

PHOBOS, PRL 104 062301 (2010)

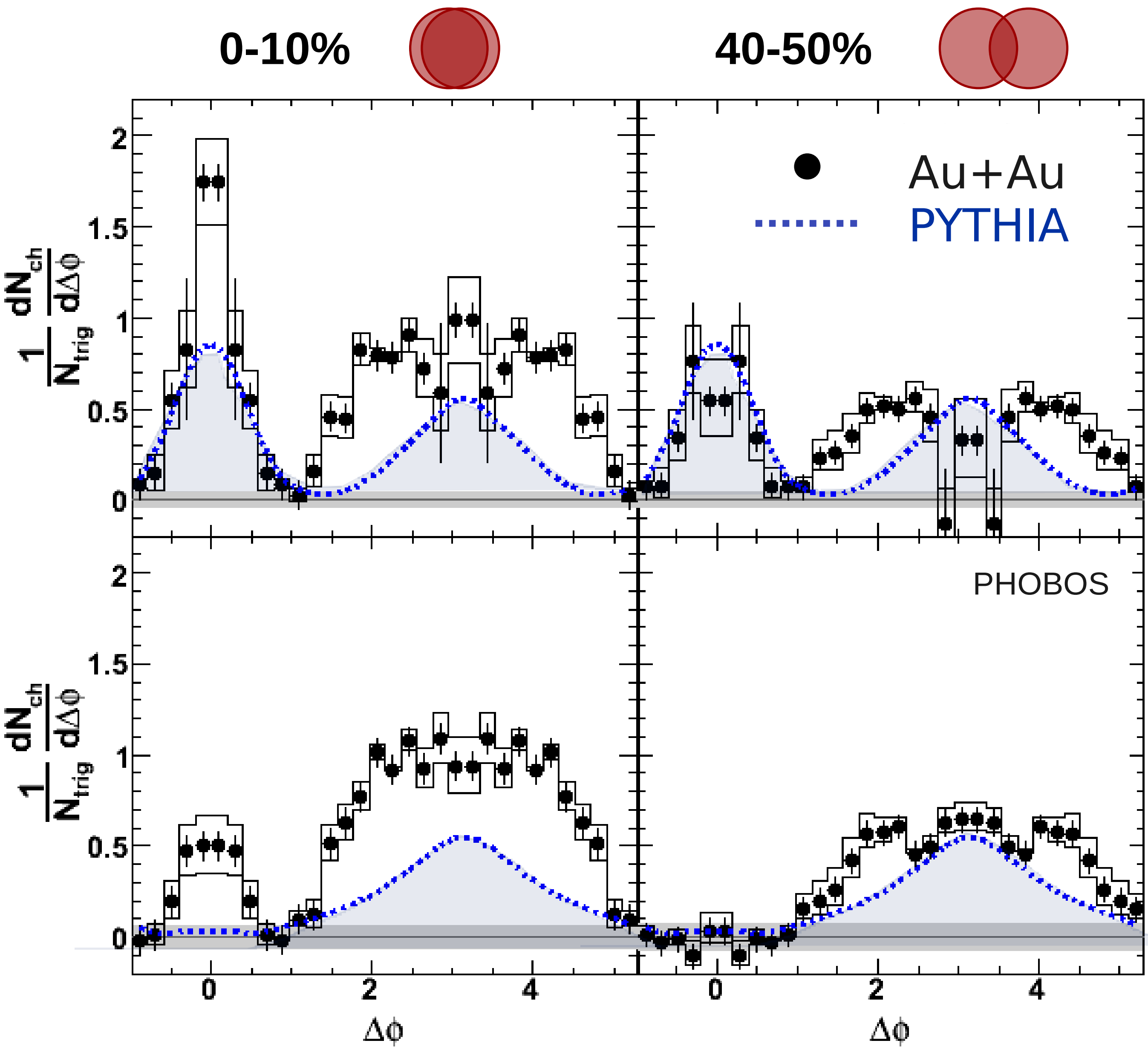
# Projected correlation along Δφ




Short-range
|Δη| <1



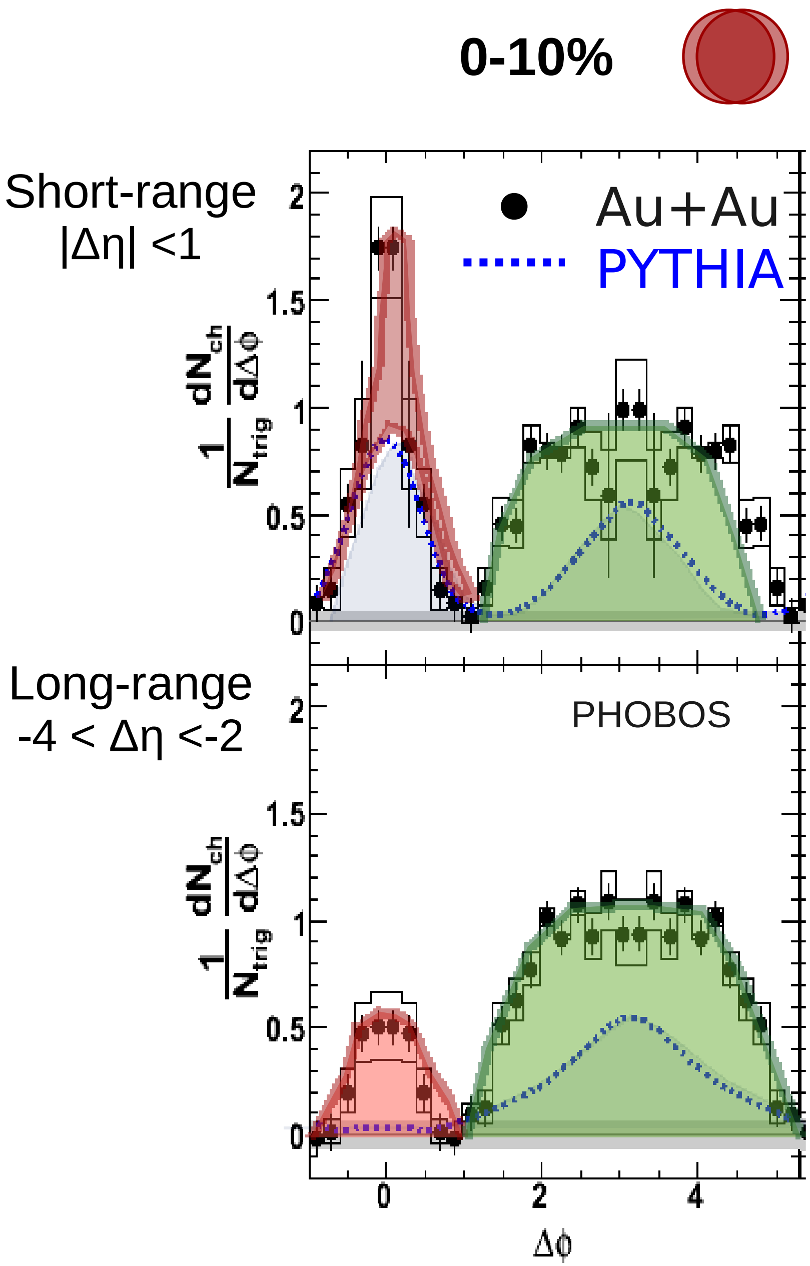

Long-range
-4 < Δη < -2



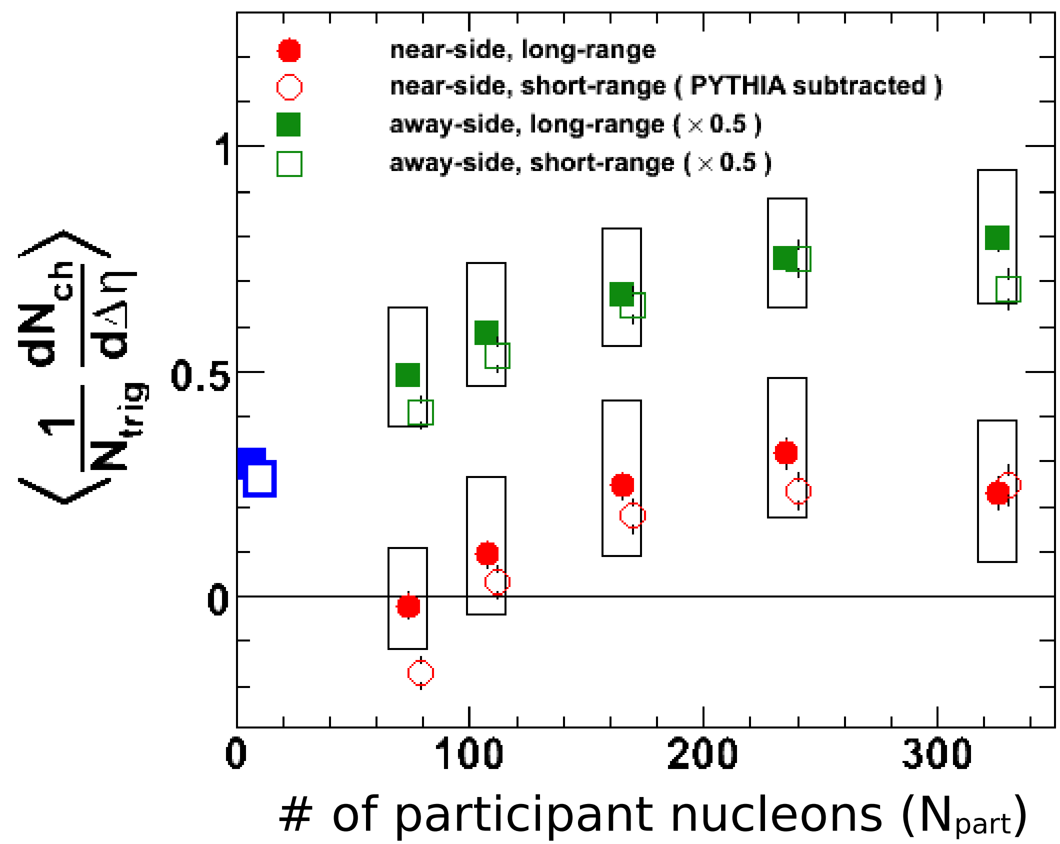

PHOBOS, PRL 104 062301 (2010)

# Integrated ridge yield (away side)






NEAR side
○ short-range minus PYTHIA
● long-range
Away side times 0.5
□ ■ short, long minus PYTHIA
□ ■ short, long range PYTHIA



# Summary



- Strong short-range correlations are observed in A+A collisions. Large sizes of clusters cannot be attributed to low mass resonances.
  - Cluster size and width scales with the fractional cross section
- Understanding of flow, non-flow correlations, flow and eccentricity fluctuations has converged
  - Short-range non-flow correlations contribute about ~10% (absolute) to $v_2$ fluctuations measurement

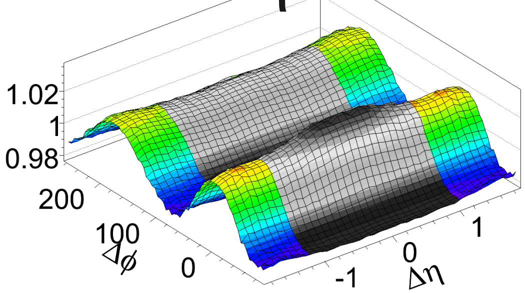
    See next slides

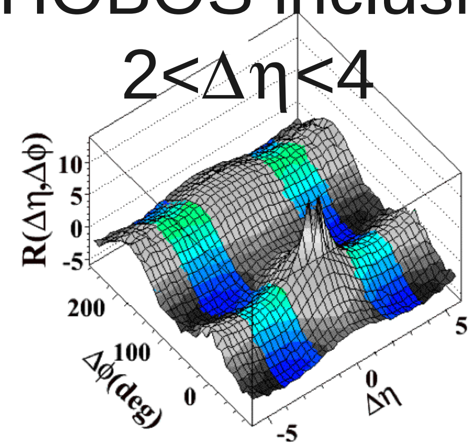
    - ~~Remaining caveat is role of long-range non-flow contribution~~
  - Initial state eccentricity fluctuations (if present) are consistent with the data, but do not leave much room for increase of fluctuations in later stages of the collision evolution
- Near-side correlations of associated particles with high $p_T$>2.5 GeV/c particle extend 4 units in pseudo-rapidity and diminish for a system with $N_{part}$~80.

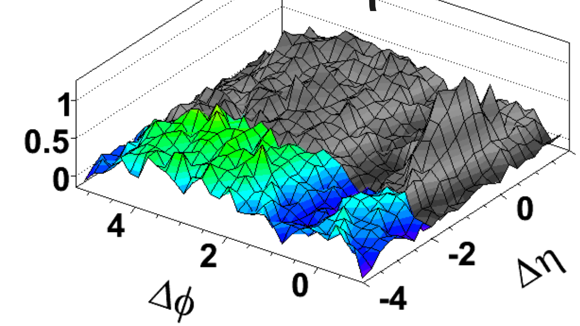
# Correlations at large $\Delta\eta$

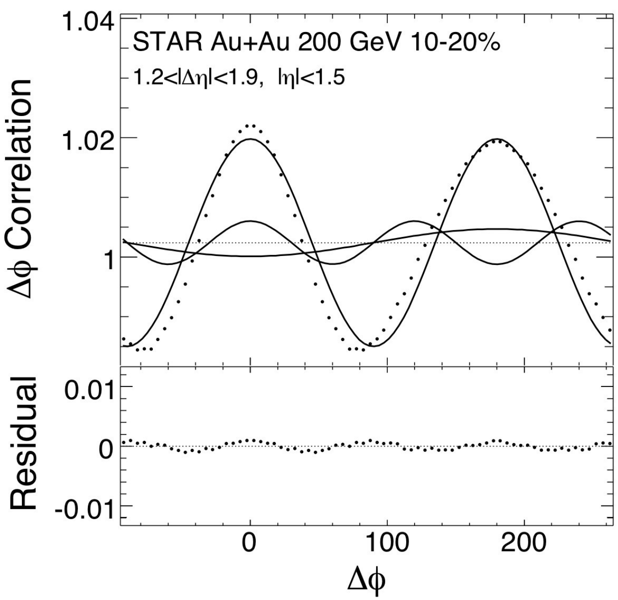


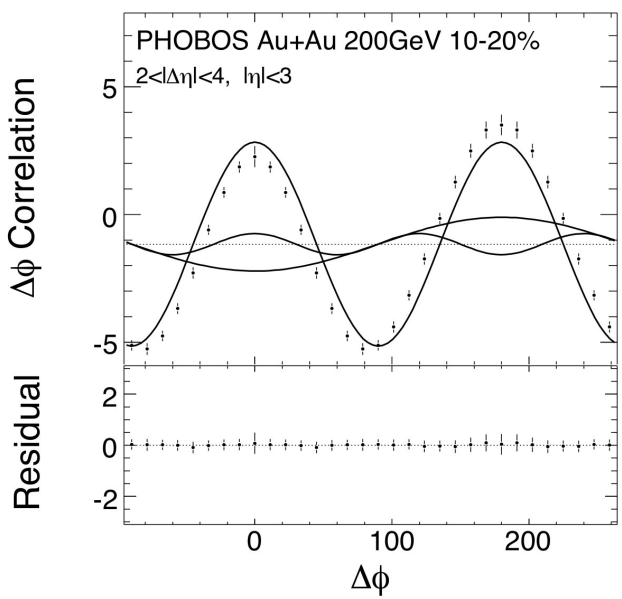
| STAR inclusive $1.2<\Delta\eta<1.9$ | PHOBOS inclusive $2<\Delta\eta<4$ | PHOBOS $p_T^{trig}>2GeV$ $2<\Delta\eta<4$ |
|---|---|---|

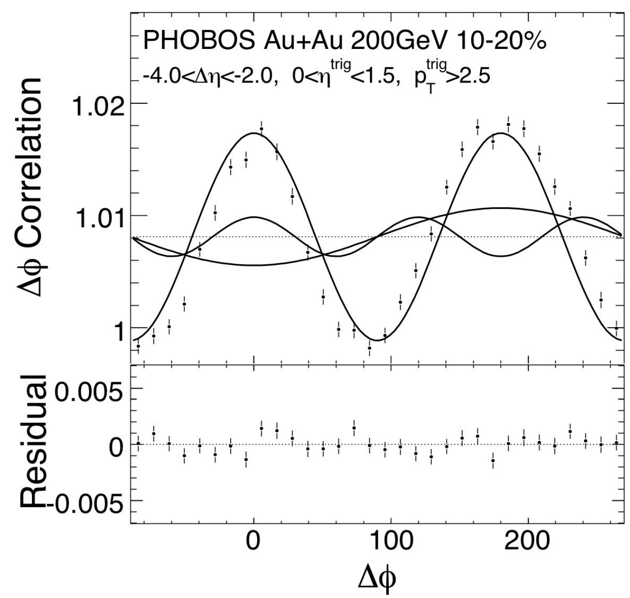



Long range correlations are well described by 3 Fourier components

Alver, Roland, arXiv:1003.0194 [nucl-th]

# Flow vs non-flow

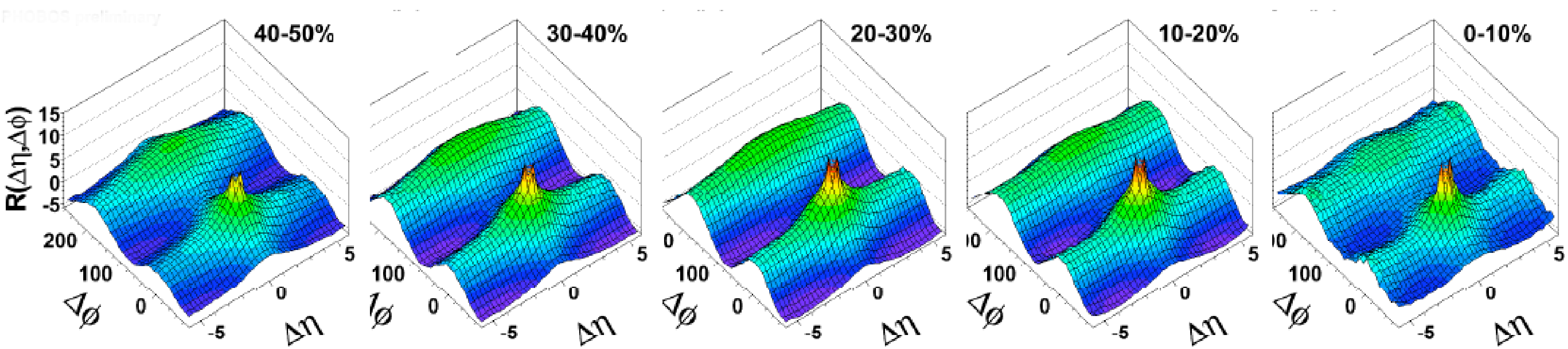


Standard picture:

- Flow = global (“collective”)
  - Second Fourier coefficient
- Non-flow = local (“clusters”)
  - All Fourier coefficients

Why is Second Fourier special?

- It is a large effect
- It is present at large Δη
- It is a function of η
- $v_2/\varepsilon$, $v_2(p_T)$, $v_2(RP)$, $v_2\{4\}$, fluctuations, etc., make “sense”

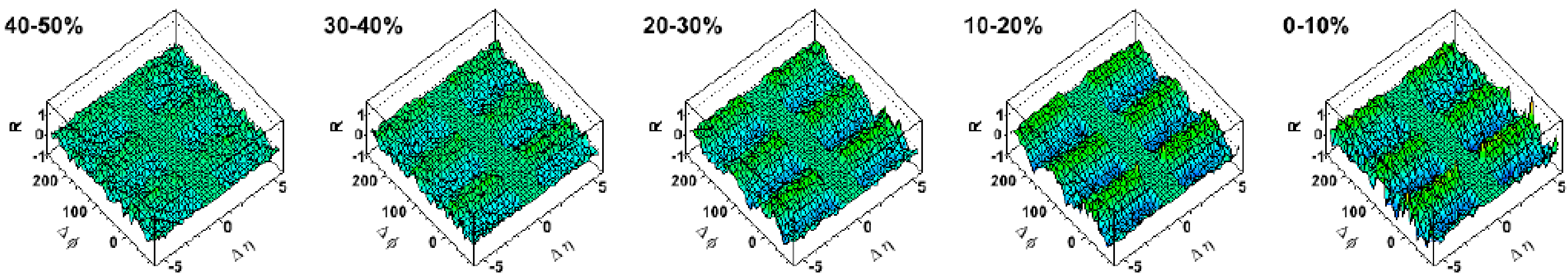


# Closer look at “non-flow”

Remove first and second Fourier contribution and suppress short-range peak (|Δη|<1)

## Is Third Fourier special?

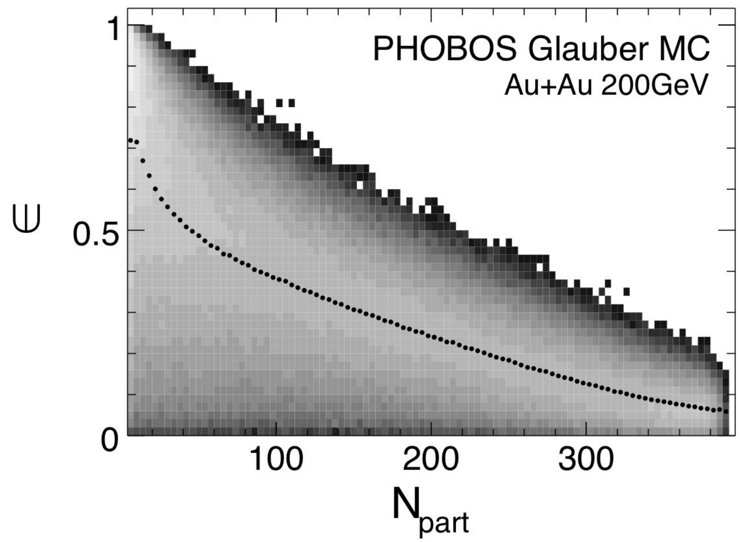
- It is a large effect
- It is there at large Δη
- Can it be linked to initial state?
- Is it a function of η (?)
- Measure centrality + $p_T$ dependence, 3 particle correlations, non-flow, etc.

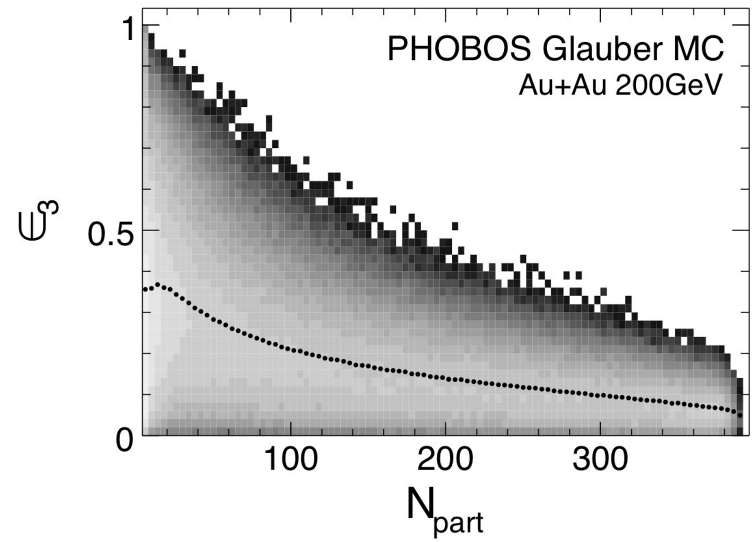



Ridge and broad away side: Even without trigger particle and at large Δη

# Participant triangularity

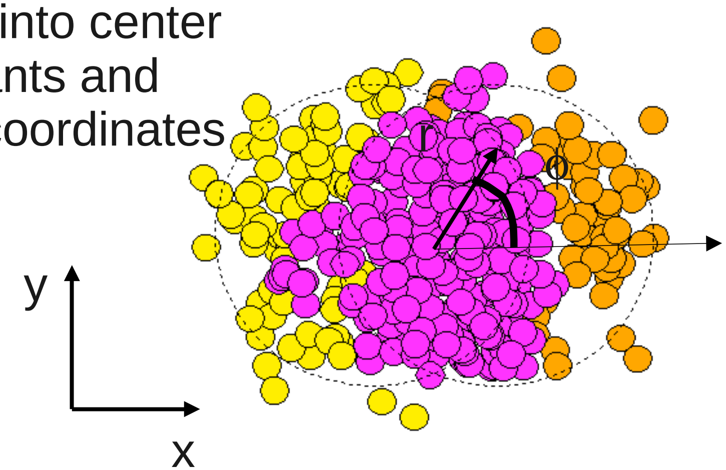
$$\epsilon_{part} \equiv \epsilon_2 = \frac{\sqrt{\langle r^2 \cos(2\phi)\rangle^2 + \langle r^2 \sin(2\phi)\rangle^2}}{\langle r^2 \rangle} \qquad \epsilon_3 = \frac{\sqrt{\langle r^2 \cos(3\phi)\rangle^2 + \langle r^2 \sin(3\phi)\rangle^2}}{\langle r^2 \rangle}$$



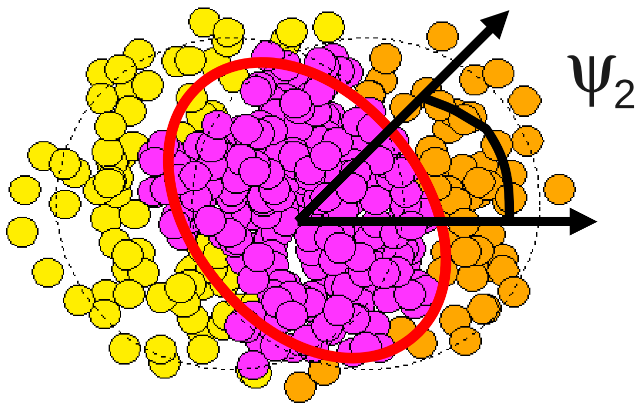

Generalize from participant eccentricity to participant triangularity

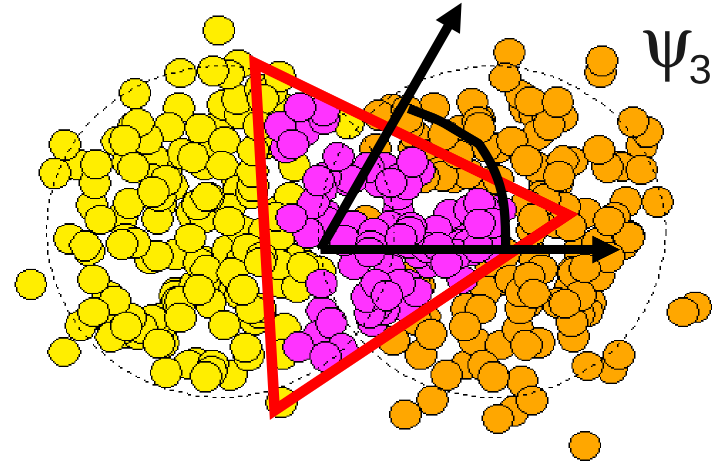
Transform into center of participants and use polar coordinates

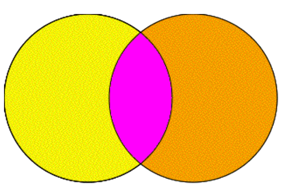



Alver, Roland, arXiv:1003.0194 [nucl-th]

# Triangular flow



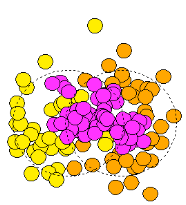



$$\frac{dN}{d\phi} \sim 1 + \sum 2 v_n \cos(n\phi - n\psi_R)$$

$$v_2 = \langle \cos(2\phi - 2\psi_R) \rangle$$

$$v_3 = 0$$



$$\frac{dN}{d\phi} \sim 1 + \sum 2 v_n \cos(n\phi - n\psi_n)$$

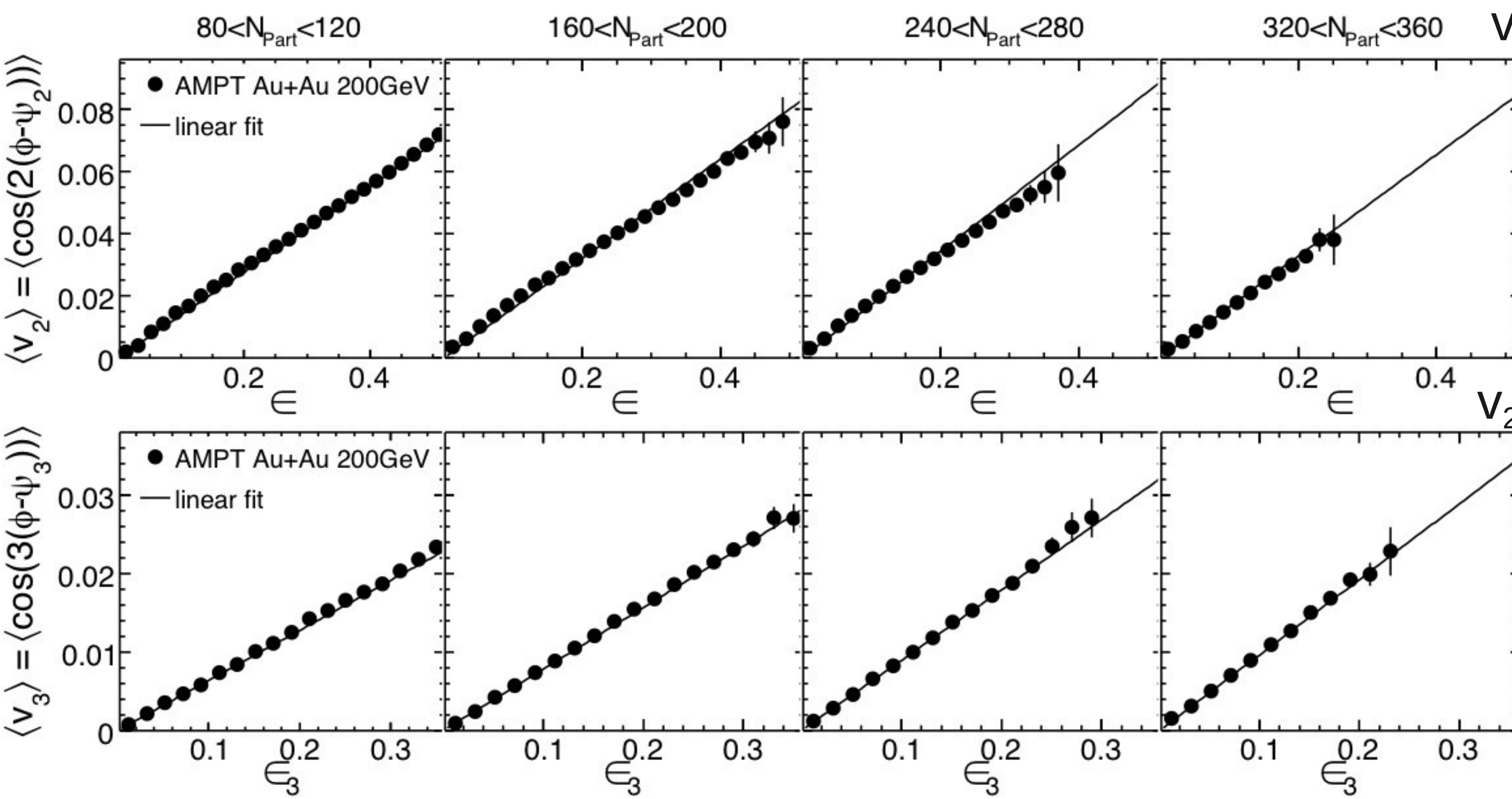
$$v_2 = \langle \cos(2\phi - 2\psi_2) \rangle$$

$$v_3 = \langle \cos(3\phi - 3\psi_3) \rangle$$

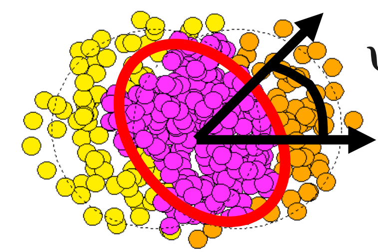
Alver, Roland, arXiv:1003.0194 [nucl-th]

(alternatively keep the sin terms)

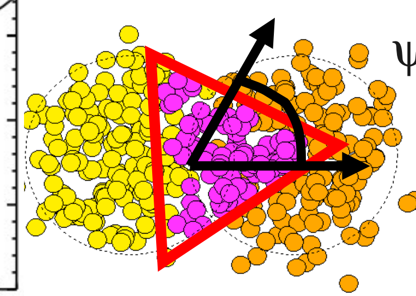
# Triangular flow in AMPT




$v_2 = \langle \cos(2\phi - 2\psi_2) \rangle$



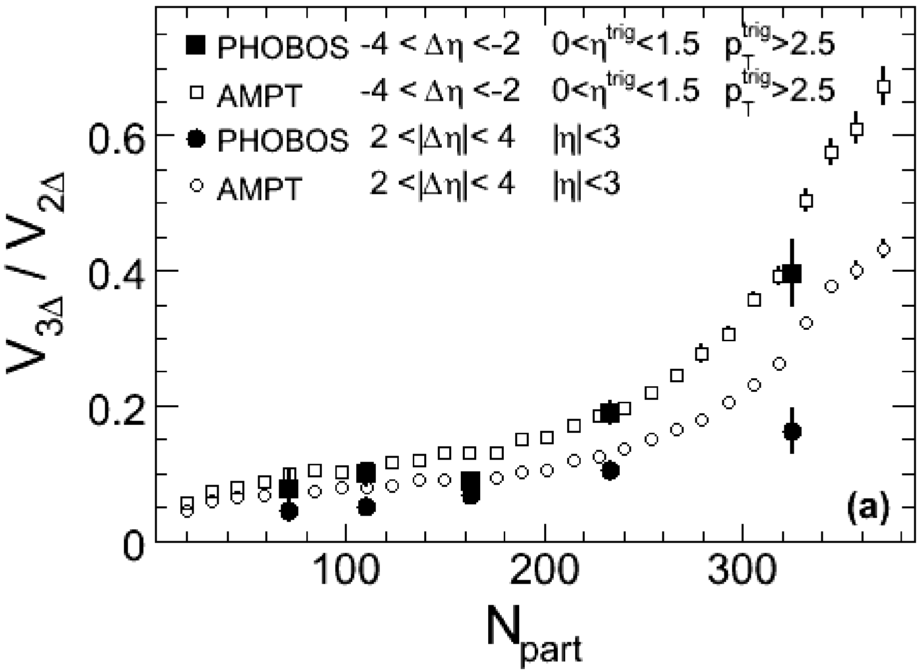

$v_2 = \langle \cos(3\phi - 3\psi_3) \rangle$



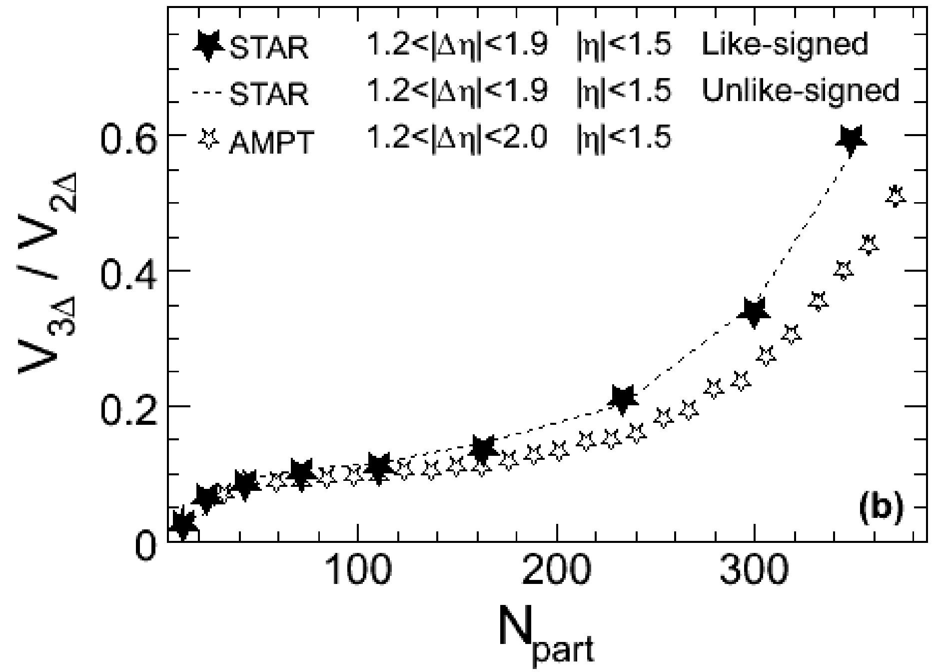

Participant triangularity leads to triangular flow in AMPT

Alver, Roland, arXiv:1003.0194 [nucl-th]

PHOBOS




STAR




The ratio of triangular to elliptic flow qualitatively agrees in data and AMPT

STAR arXiv:0806.0513
PHOBOS PRC 81, 024904 (2010)
PHOBOS PRL 104, 06230 (2010)

Alver, Roland, arXiv:1003.0194 [nucl-th]

# Summary/Outlook

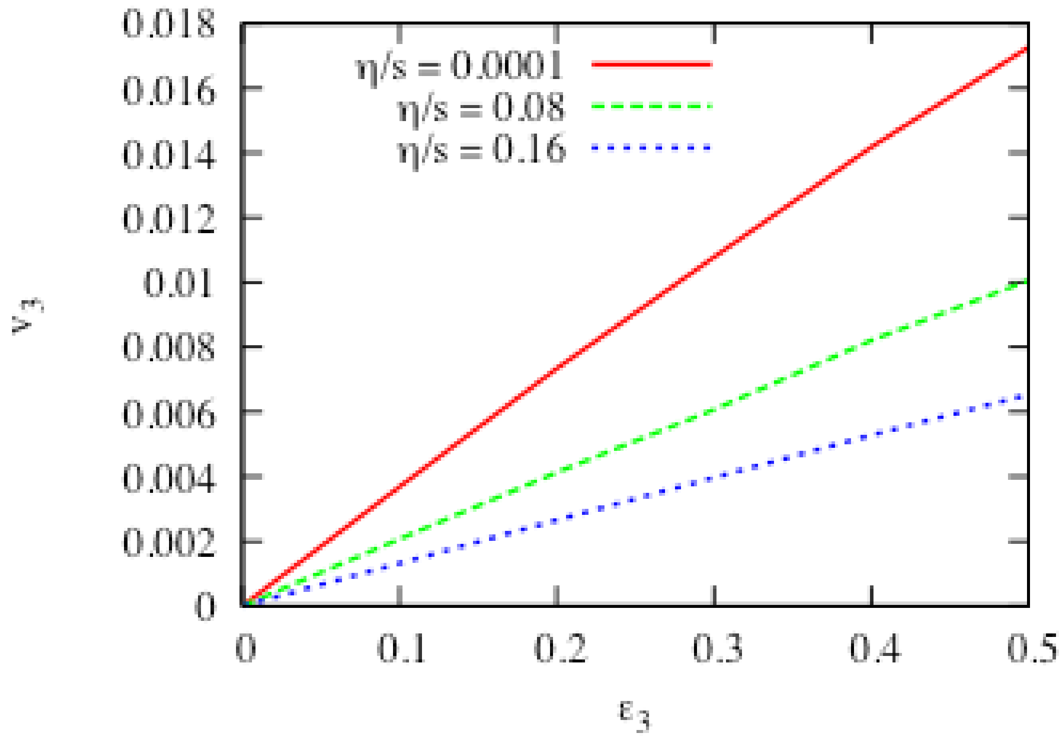


- Triangular flow (caused by initial state fluctuations) is a natural explanation for the ridge effects.
  - Need to (experimentally) characterize its properties
    - Centrality and η (and $p_T$) dependence
    - Understand non-flow contribution
- If collective property, then it must be treated as background to all our two (and three) particle correlation measurements, ie taken out as we do for second Fourier component
- What about higher moments? Yes, all moments are there in the initial state, but the question is: Are (or how often are) they dominating the initial pressure gradients
- Theory side is already working on establishing the connection between hydro and triangularity

# Extra

# Triangular flow in hydro



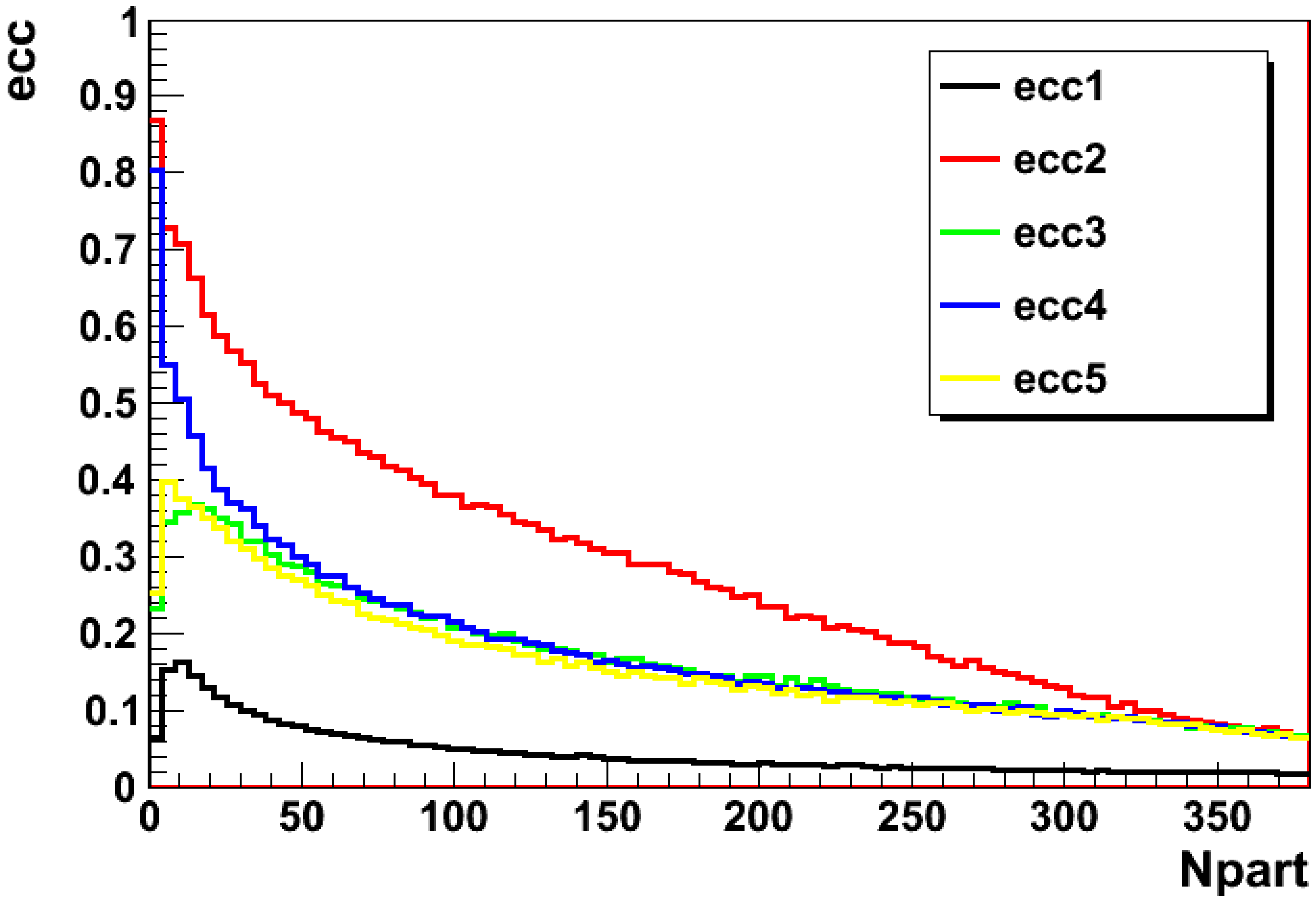



Triangular flow may be a new handle on the initial geometry and the hydrodynamic expansion of the medium

Luzum, Ollitrault (work in progress)

# Generalized eccentricity

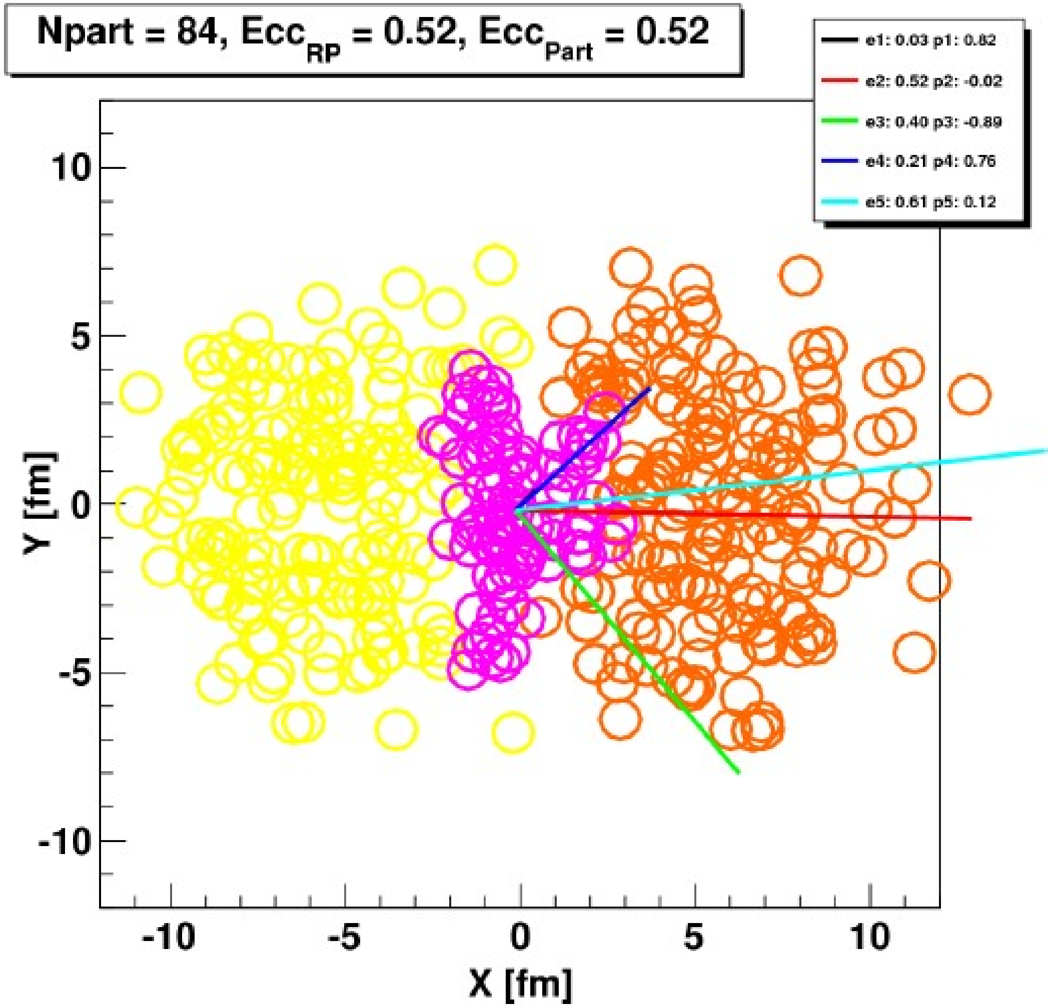




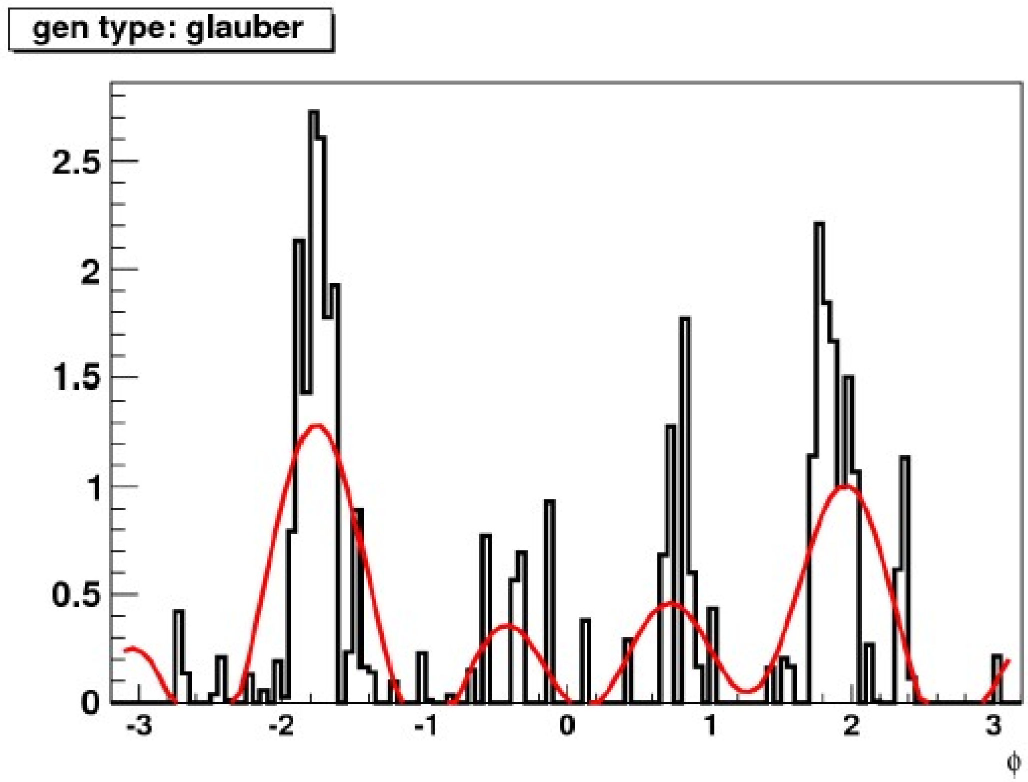

Some work in progress figures

# Example for “Pentagrularity”





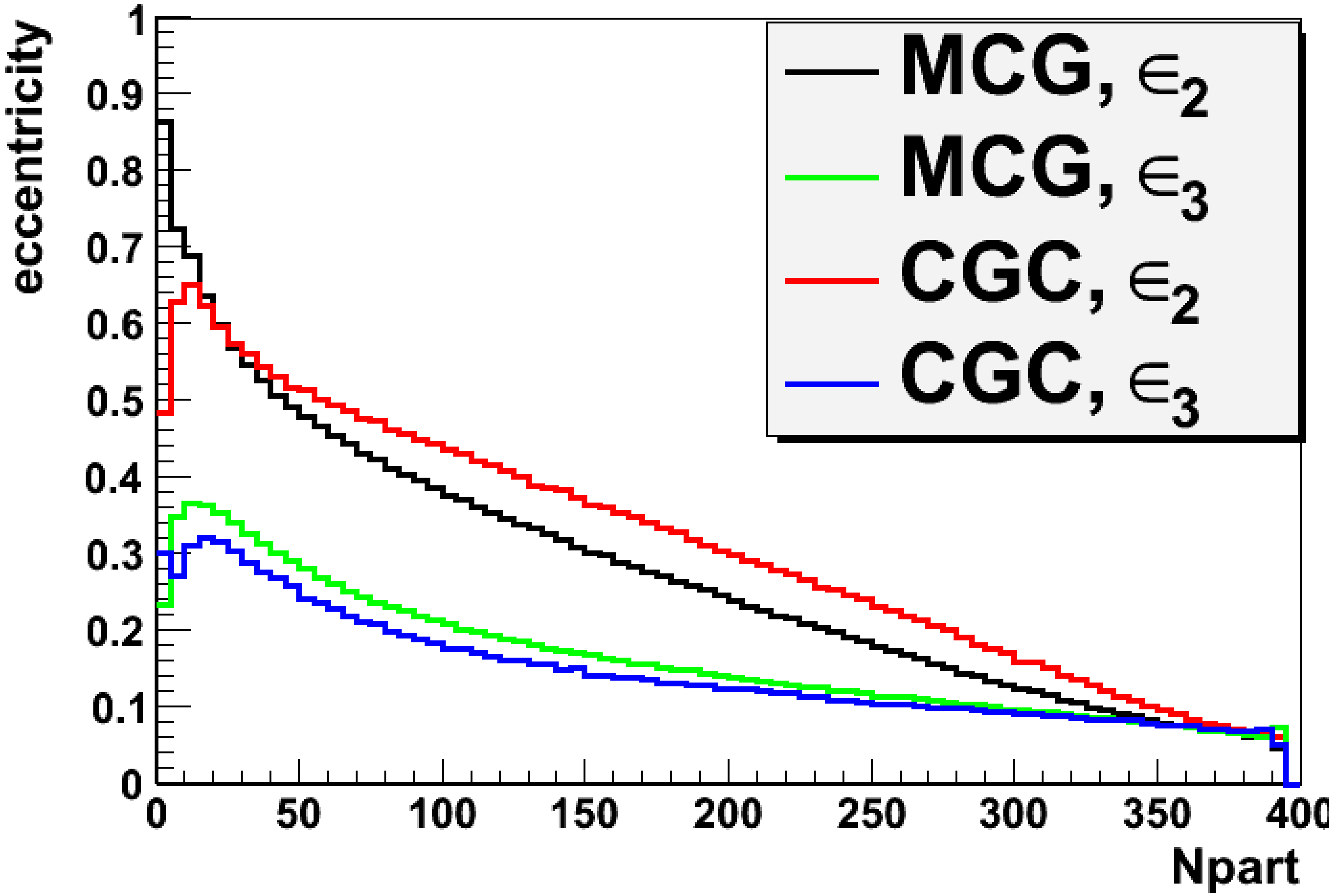

Ecc5 > 0.6
(but note also
ecc2~0.5 and ecc3~0.4)




Some work in progress figures

# Eccentricity and triangularity



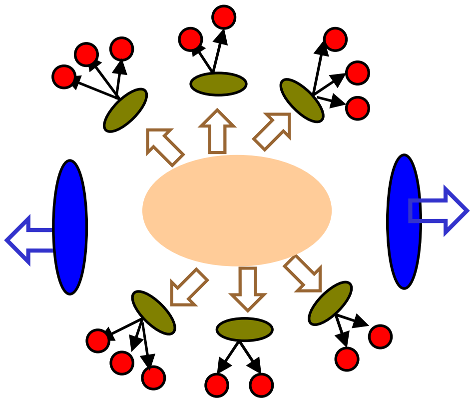



Some work in progress figures

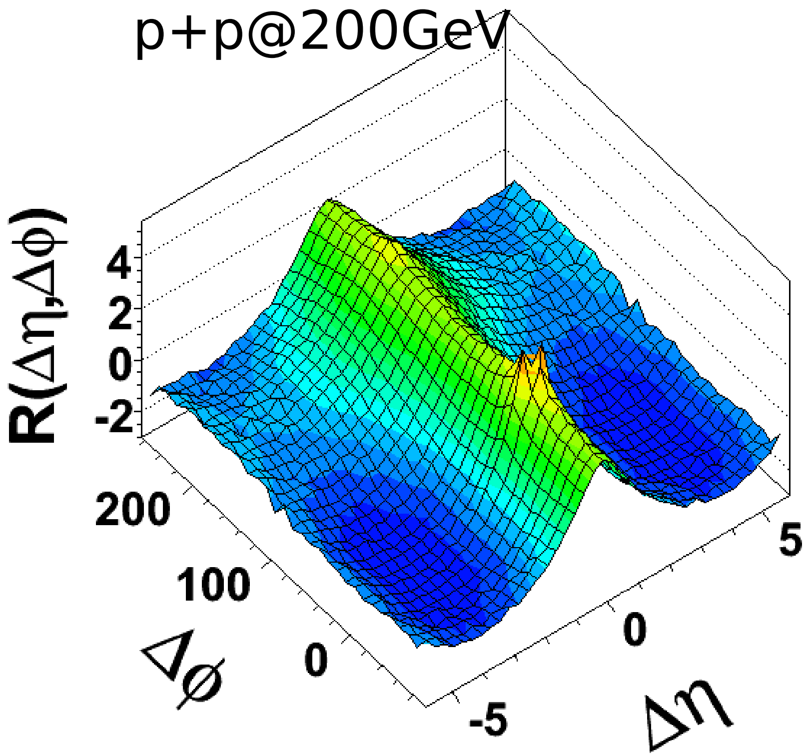
# Cluster model



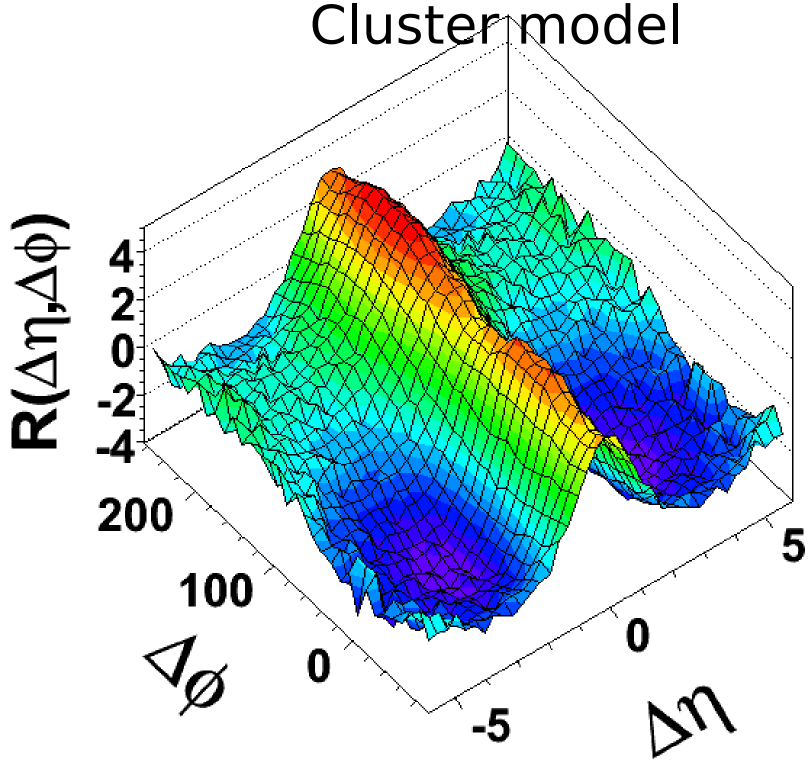
Possible source for correlations:
Production of intermediate objects
which decay into particles



p+p@200GeV



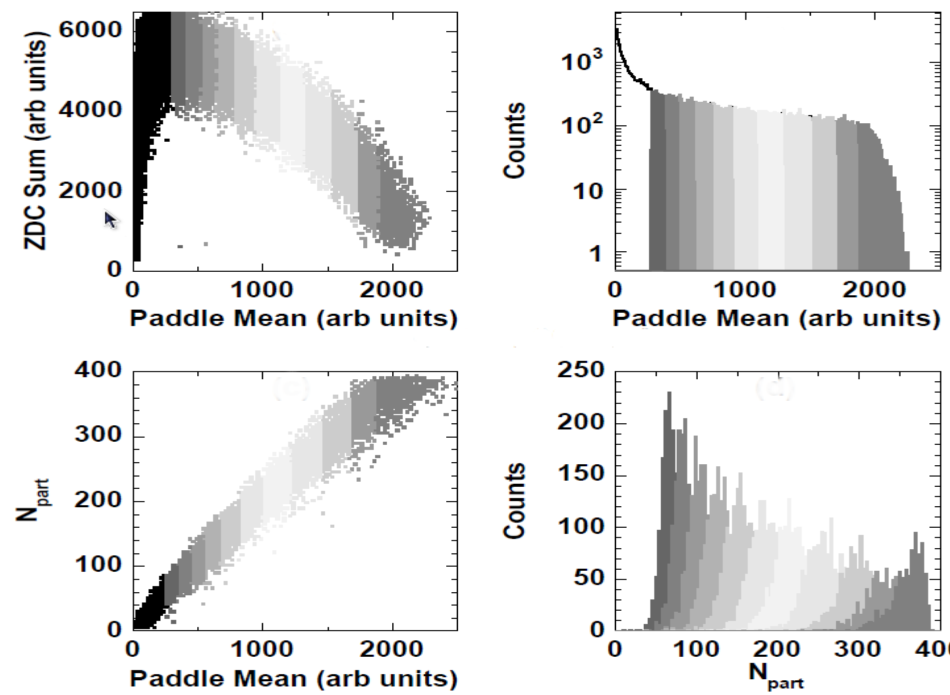

Cluster model

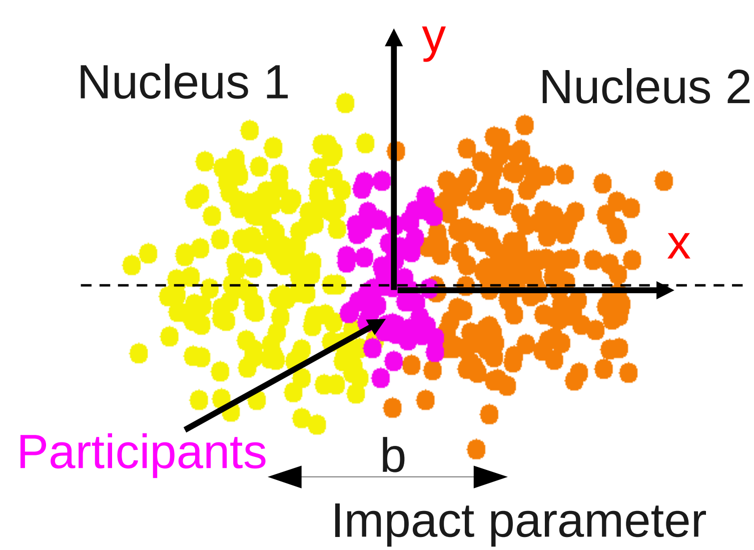


# Centrality determination



- Makeup of nuclei
  - Made up of nucleons drawn from Wood-Saxon distribution
  - Separate by b (with dN/db~b)
- Collision of nuclei
  - Assume: Nucleons travel along z on straight-line paths and interact when their centers are within $\sqrt{\sigma_{inel}^{NN}/\pi}$
  - #Participants is number of nucleons that interact at least once ($N_{part}$~A)
  - #NN-collisions is total number of collisions ($N_{coll}$~$A^{4/3}$)
- Relate to data via Glauber MC based detector simulations




PHOBOS, NPA 757 28 (2005)

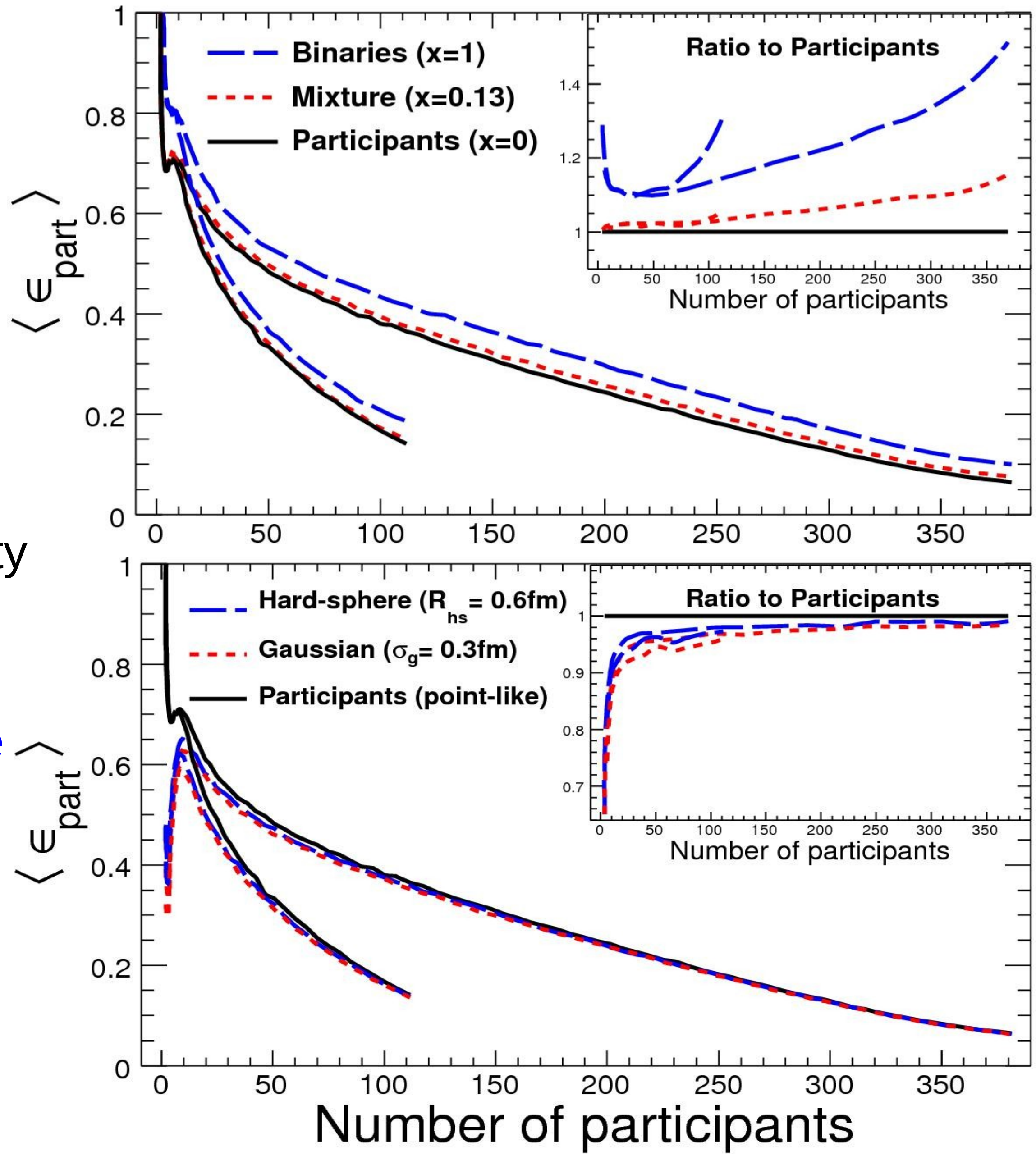
# Assumptions of particle production

- Model two component scenario
  - Matter production via participants and binary collisions

$$\frac{dN^{AA}}{d\eta}=\frac{dN^{pp}}{d\eta}\left(\frac{1-x}{2}N_{part}+x\,N_{coll}\right)$$

  - Mixture with x=0.13 describes mid-rapidity dN/dη quite well
    - 10% increase in eccentricity for central Au+Au
- Include thermalization time by smearing the matter around the original production point
  - Hard-sphere and Gaussian
    - For chosen set of parameters only a very small effect

NB: More generalized studies also done, see Broniowski et al., PRC 76 (2007) 054905




PHOBOS+Heinz, PRC 77 (2008) 014906

Two-particle cumulant

$$v\{2\}=\sqrt{\langle\cos(\phi_1-\phi_2)\rangle}$$

Measures:

$$v\{2\}^2=\langle v\rangle^2+\sigma_{v_2}^2+\delta$$

$$v\gg 1/\sqrt{M}$$

Four-particle cumulant

$$v\{4\}=\left(2\langle\cos(\phi_1-\phi_2)\rangle^2-\langle\cos(\phi_1+\phi_2-\phi_3-\phi_4)\rangle\right)^{1/4}$$

Measures:

$$v\{4\}^2=\langle v\rangle^2-\sigma_{v_2}^2$$

$$v\gg 1/M^{3/4}$$

$$v\{subEP\}=\frac{\langle\cos(\phi-\psi_A)\rangle}{R}$$

$$R=\sqrt{\langle\cos(\psi_A-\psi_B)\rangle}$$

Measures:

$$v\{subEP\}^2=\langle v\rangle^2+\left(1-f(R)\right)\sigma_{v_2}^2$$
$$+\left(1-2f(R)\right)\delta$$

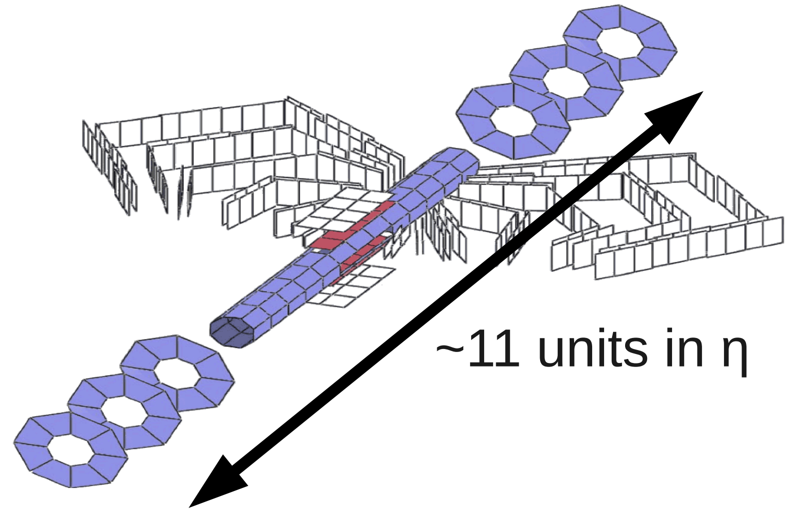
NB: For simplicity, n (as index and in cos terms) dropped

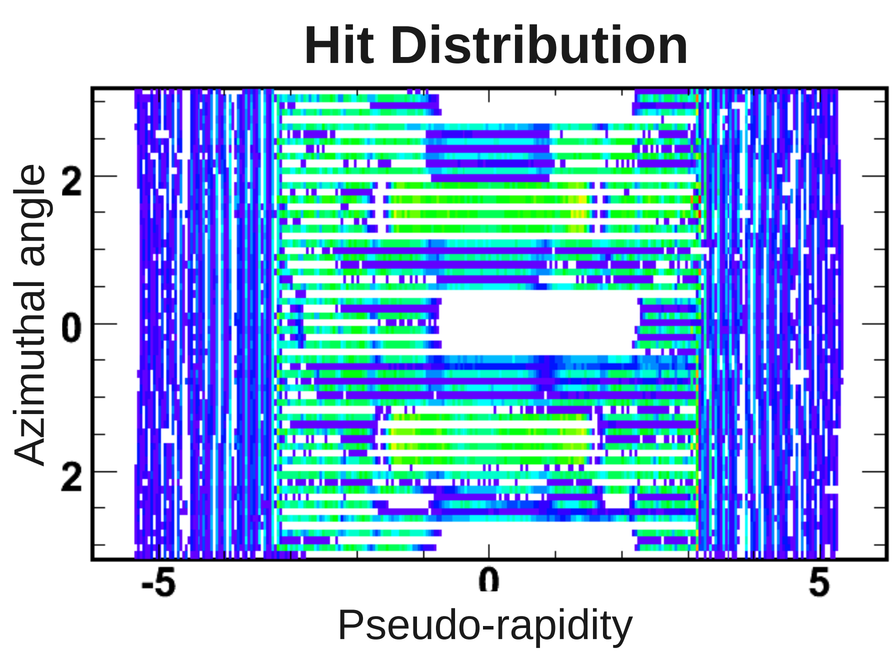
Ollitrault, Poskanzer, Voloshin
PRC 80 80 014904 (2009)

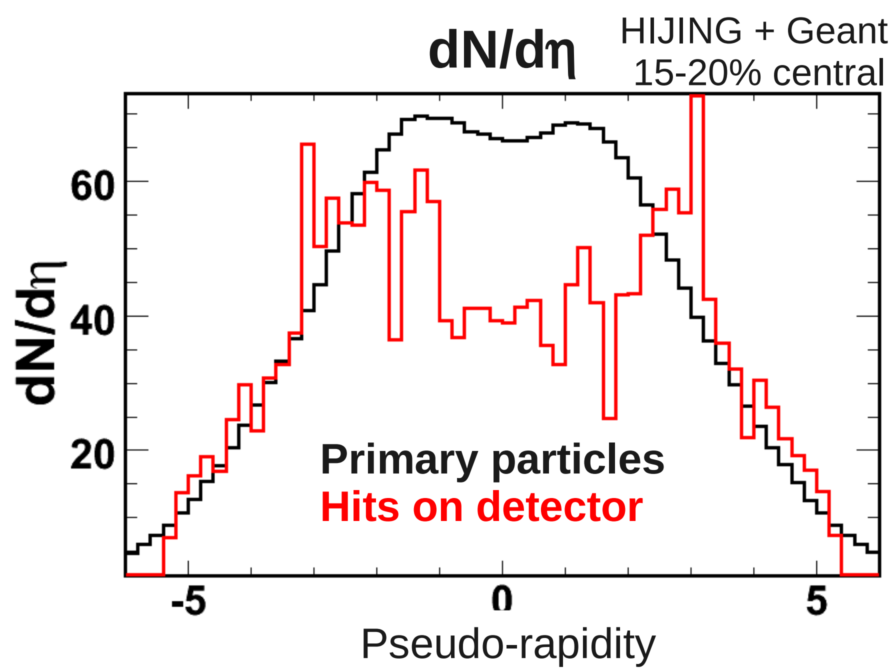
# Challenges of event-by-event $v_2^{obs}$



- PHOBOS Multiplicity Array
  - $-5.4<\eta<5.4$ coverage
  - Holes and granularity differences
- Usage of all available information in event to determine event-by-event a single value for $v_2^{obs}$



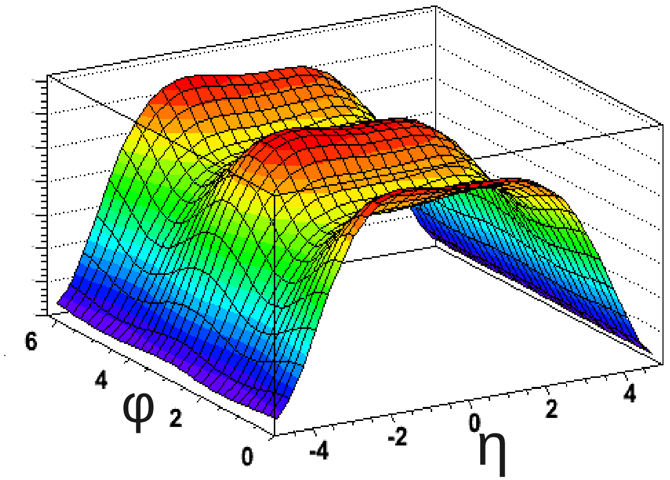



# Event-by-event measurement of $v_2^{obs}$

- Event-by-event measurement of $v_2^{obs}$
  - Deal with acceptance effects
  - Use all available hit information

**Probability distribution function**




- Probability distribution function for hit positions:

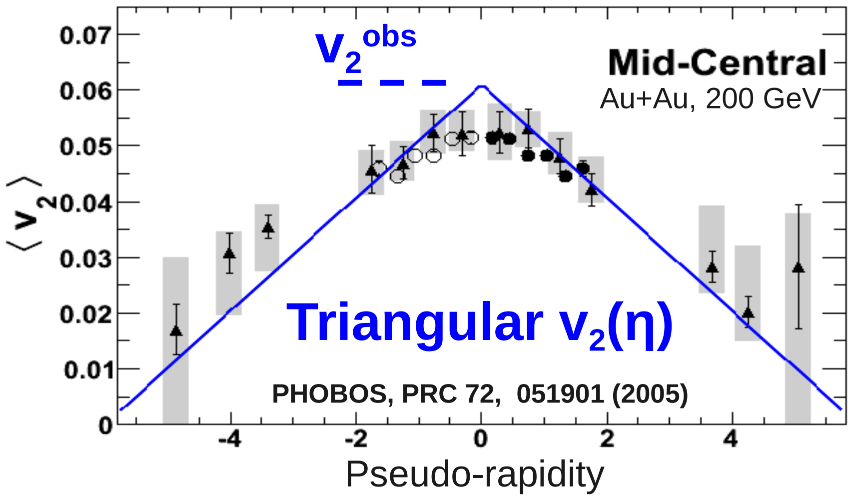
$$P(\eta,\phi;\, v_2^{obs},\phi_0) = \underbrace{p(\eta)}_{\text{Normalization incl. acceptance}} \underbrace{[1+2v_2(\eta)\cos(2\phi-2\phi_0)]}_{\text{Probability of hit in }(\varphi,\eta)}$$

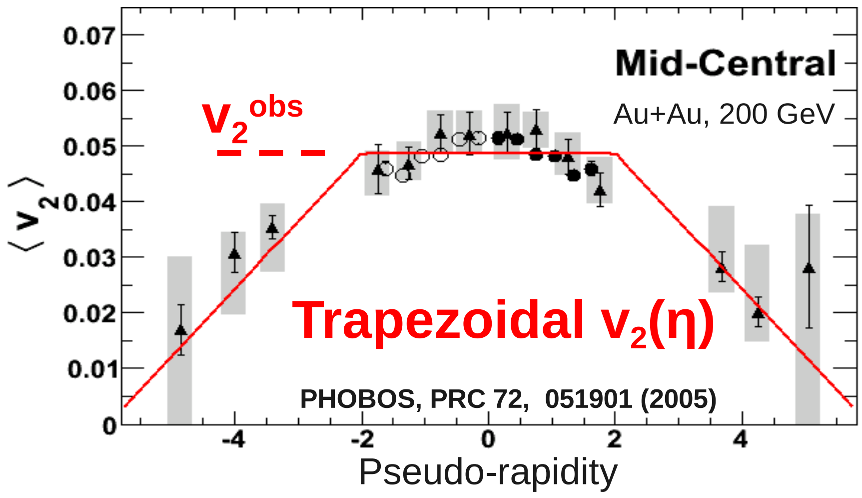
- Maximize the likelihood function to obtain $v_2^{obs}$ and $\varphi^0$ (event plane angle)

$$L(v_2^{obs},\, \phi_0) = \prod_{i=1}^{n} P(\eta_i,\phi_i;\, v_2^{obs},\phi_0)$$

PHOBOS, PRL 104 142301 (2010)

# Event-by-event measurement of $v_2^{obs}$




$$P(\eta,\phi;\, v_2^{obs},\phi_0)=p(\eta)[1+2\underline{v_2(\eta)}\cos(2\phi-2\phi_0)]$$

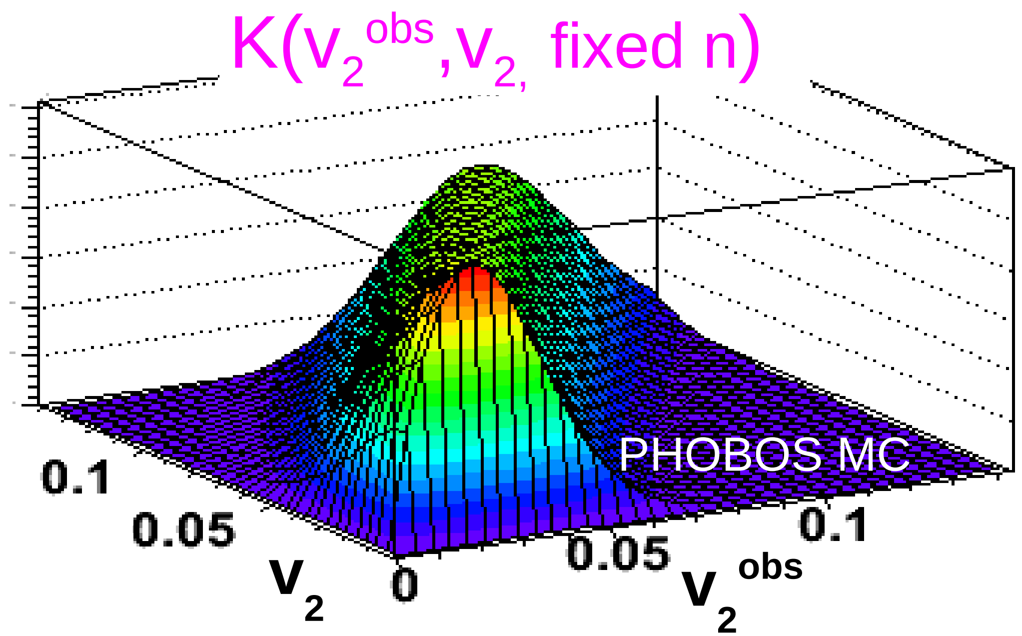
Use known, measured shape

Analysis is run on triangular and trapezoidal shape.
Results are averaged at the end.

PHOBOS, PRL 104 142301 (2010)

# Determining the kernel

- "Measure" and record the $v_2^{obs}$ distribution in bins of $v_2$ and multiplicity (n) from large MC samples
  - $1.5{\cdot}10^6$ HIJING events
  - Modified φ to include triangular or trapezoidal flow




- Fit response function (ideal case)

$$K(v_2^{obs}, v_2, n) = \frac{v_2^{obs}}{\sigma^2} e^{-\left(\frac{v_2^{obs\,2} + v_2^2}{2\sigma^2}\right)} I_0\left(\frac{v_2^{obs} v_2}{\sigma^2}\right)$$

(J.-Y.Ollitrault, PRD (1992) 46, 226)

- Changed to account for detector effects

$$v_2 \rightarrow (An + B) v_2 \qquad \sigma = \frac{C}{\sqrt{n}} + D$$

(suppression) (finite resolution)

PHOBOS, PRL 104 142301 (2010)

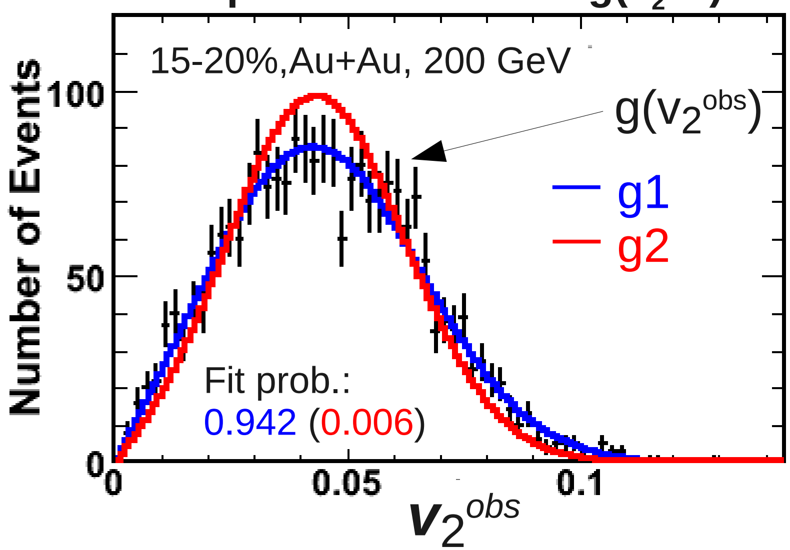
# Extracting dynamical fluctuations 

$$\underbrace{g(v_2^{obs})}_{\text{Measured}} = \int_0^1 \underbrace{K(v_2^{obs}, v_2)}_{\text{Constructed from MC}} \, f(v_2) \, dv_2$$

Gaussian Ansatz:

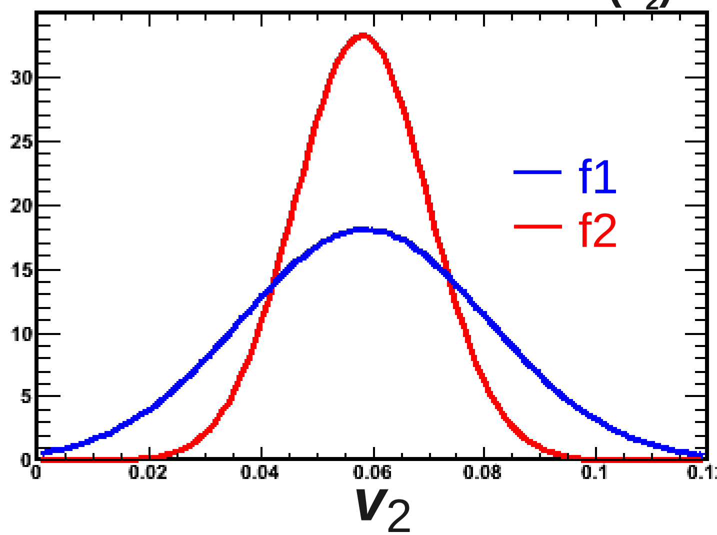
$$f(v_2) = \exp\left[\frac{-(v_2 - \langle v_2 \rangle)^2}{2\sigma_{v_2}^2}\right]$$

**Comparison with data $g(v_2^{obs})$**




Use kernel + integrate

**Different trials for Ansatz $f(v_2)$**



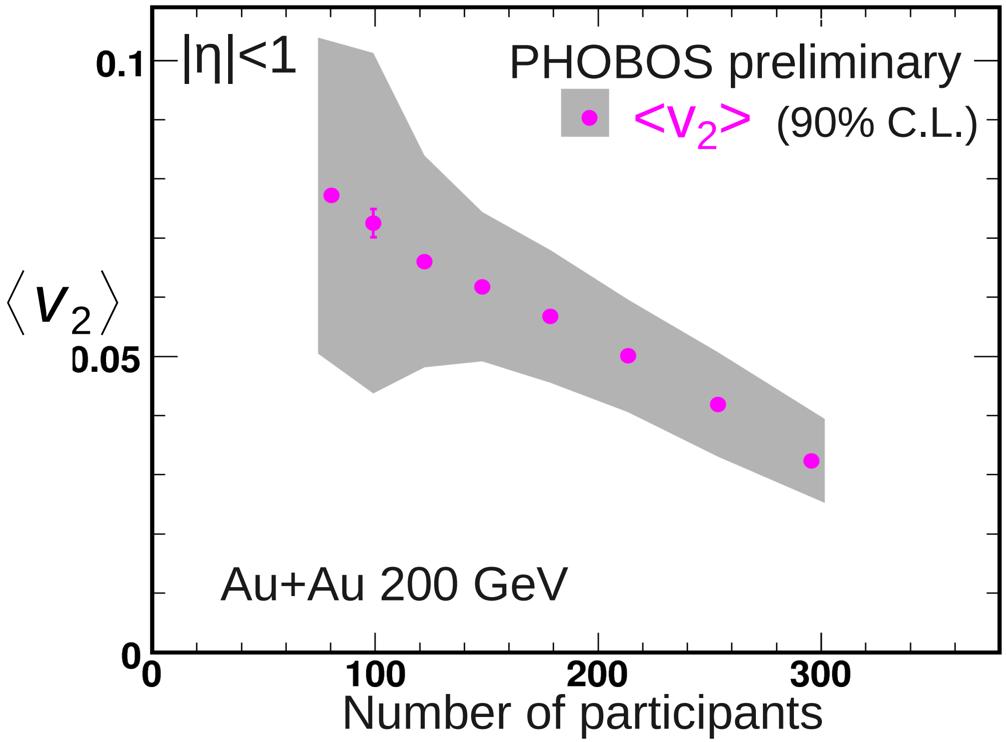

Compare expected $g(v_2^{obs})$ for trials with data:

Maximum-Likelihood fit $\rightarrow \langle v_2 \rangle$ and $\sigma_{v_2}$

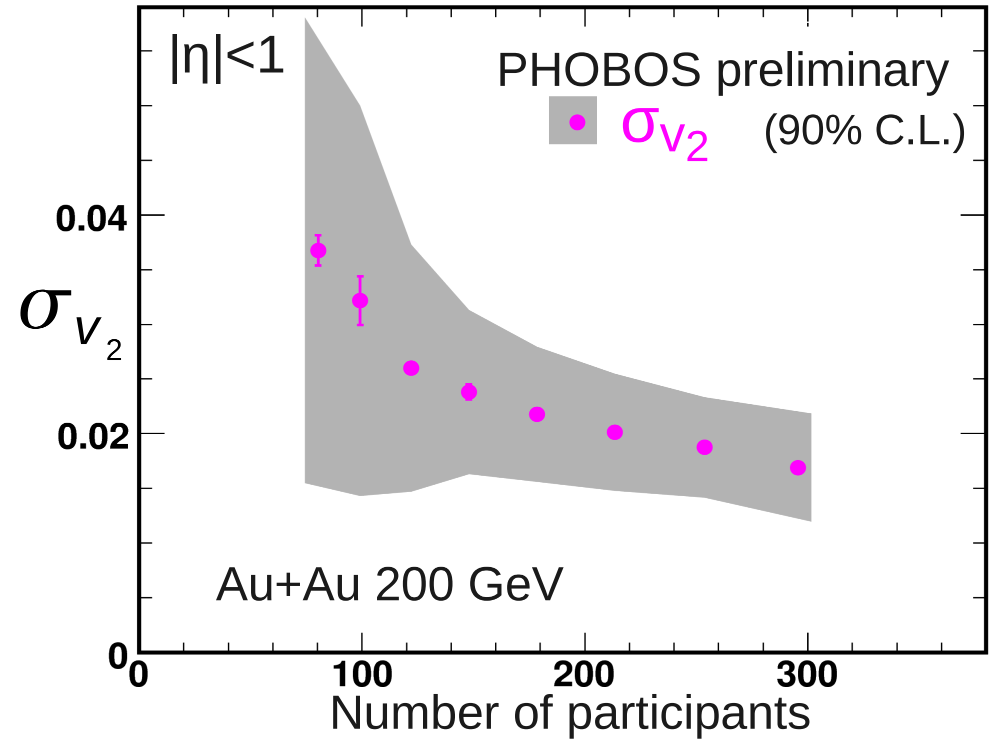
PHOBOS, PRL 104 142301 (2010)

# Elliptic flow fluctuations: $<v_2>$ and $\sigma_{v_2}$ 

Mean elliptic flow




Dynamical flow fluctuations




Systematic errors:

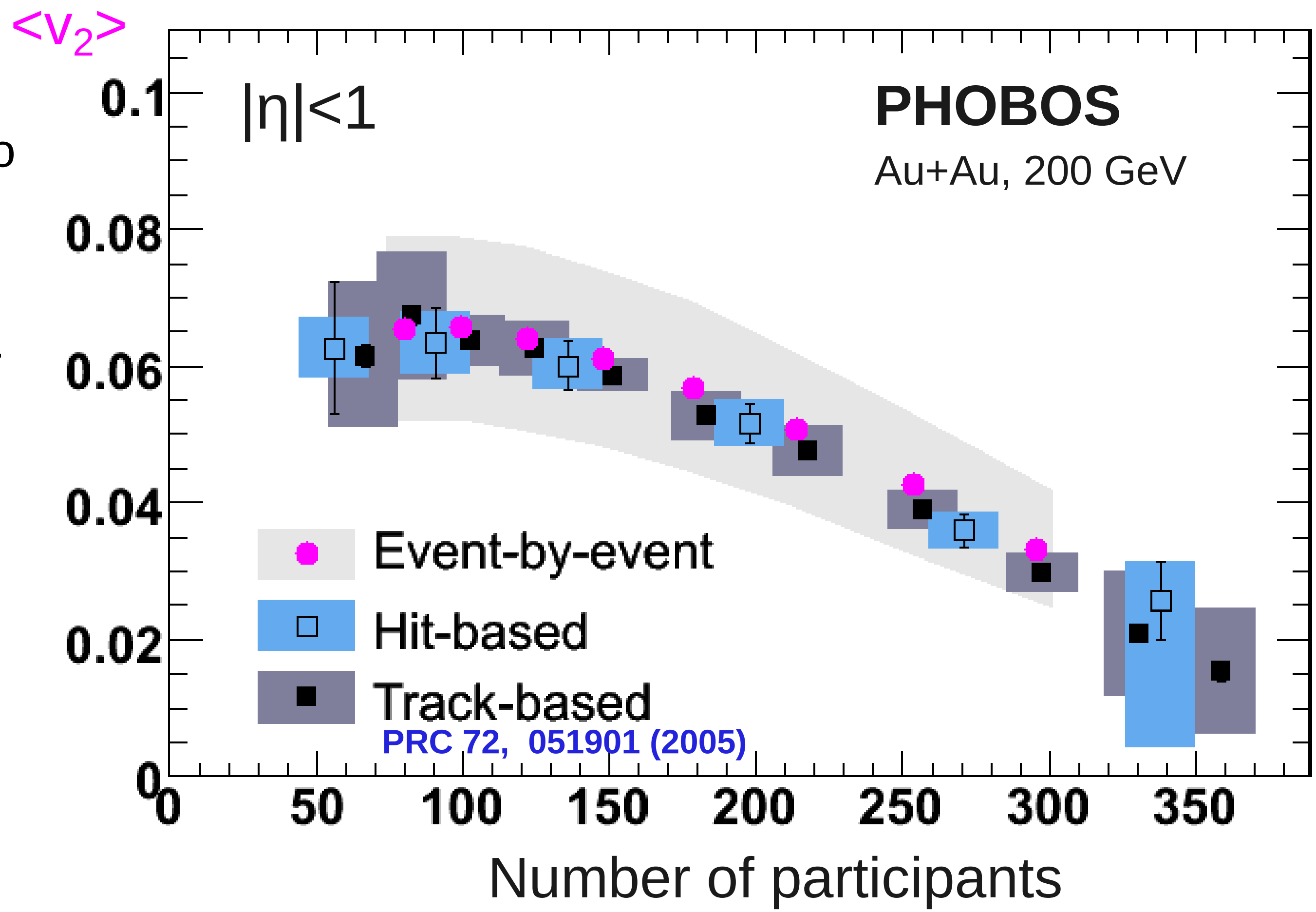
- Variation in η-shape
- Variation of $f(v_2)$
- MC response
- Vertex binning
- $\Phi_0$ binning

"Scaling" errors cancel in the ratio: relative fluctuations, $\sigma_{v_2}/<v_2>$

PHOBOS, PRL 104 142301 (2010)

# Event-by-event $v_2$ vs published results 

- Standard methods
  - Averaged over events to measure the mean
  - Hit- and track-based
  - Use reaction plane sub-event technique




Very good agreement of the event-by-event measured mean $v_2$ with the hit- and tracked-based, event averaged, published results

PHOBOS, PRL 104 142301 (2010)

# Numerical subtraction

$$K(v_2^{obs}, v_2, n) = BG(v_2^{obs}, v_2, \sigma_n), \quad \sigma_n = 1/\sqrt{2n}$$

$$K_\delta(v_2^{obs}, v_2, n) = BG(v_2^{obs}, v_2, \sqrt{\sigma_n^2 + \sigma_\delta^2}), \quad \sigma_n = 1/\sqrt{2n}, \sigma_\delta = \sqrt{\delta/2}$$

$$g(v_2^{obs}) = \int K_\delta(v_2^{obs}, v_2, n) f_{flow}(v_2)\, \mathrm{d}v_2$$

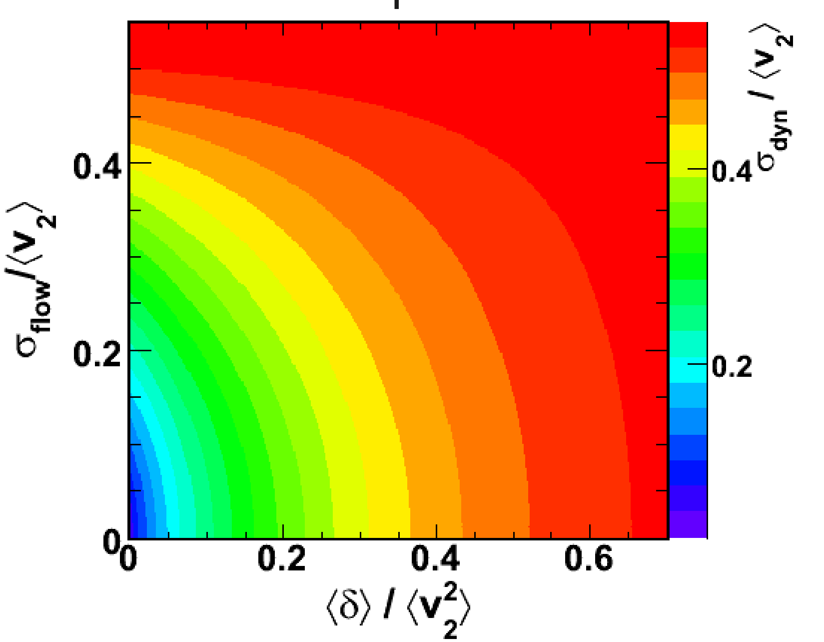
Generate $g(v_2^{obs})$ using this

$$g(v_2^{obs}) = \int K(v_2^{obs}, v_2, n) f(v_2)\, \mathrm{d}v_2$$

Do a fit using this

- Keep results as lookup table
- Results slightly depend on $\sigma_n$
  - Use $\sigma_n$ = 0.4, 0.6 and 0.8

Lookup table




PHOBOS, PRC 81 034915 (2010)

# Construction of correlated yield



$$\frac{1}{N_{trig}}\frac{d^2N_{ch}}{d\Delta\phi\, d\Delta\eta} = B(\Delta\eta)\left\{\boxed{\frac{s(\Delta\phi,\Delta\eta)}{b(\Delta\phi,\Delta\eta)}} - a(\Delta\eta)\left[1 + 2V(\Delta\eta)\cos(2\Delta\phi)\right]\right\}$$

$$\frac{s(\Delta\phi,\Delta\eta)}{b(\Delta\phi,\Delta\eta)}$$

**Raw correlation:** ratio of per-trigger same event pairs to mixed event pairs

$$1 + 2V(\Delta\eta)\cos(2\Delta\phi)$$

**Elliptic flow:** $V(\Delta\eta) = \langle v_2^{trig}\rangle\langle v_2^{assoc}\rangle$

PHOBOS Phys. Rev. C **72**, 051901(R) (2005)

$$a(\Delta\eta)$$

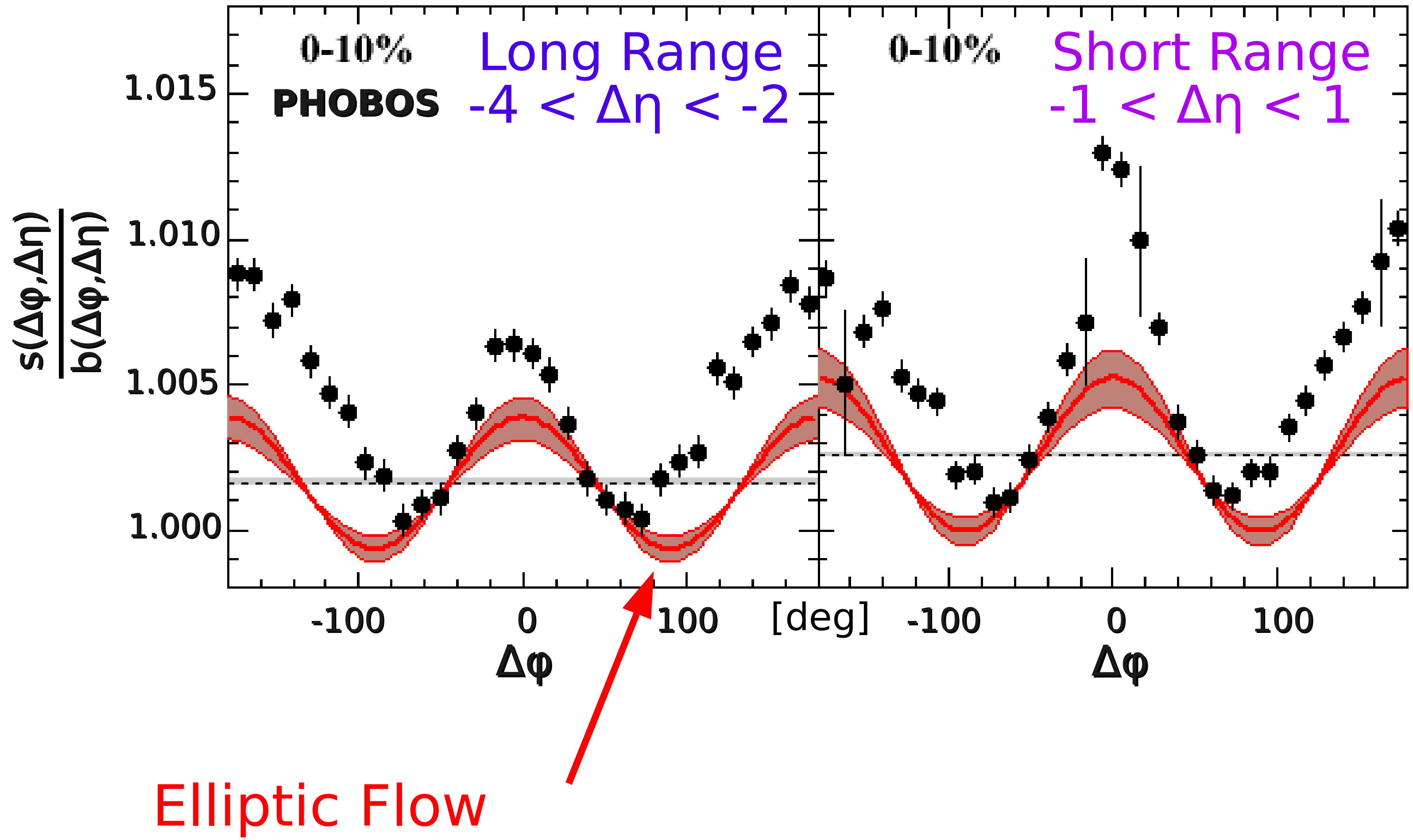
**Scale factor:** accounts for small multiplicity difference between signal and mixed events

$$B(\Delta\eta)$$

**Normalization term:** relates flow-subtracted correlation to correlated yield

PHOBOS, PRL 104 062301 (2010)

# Subtraction of elliptic flow

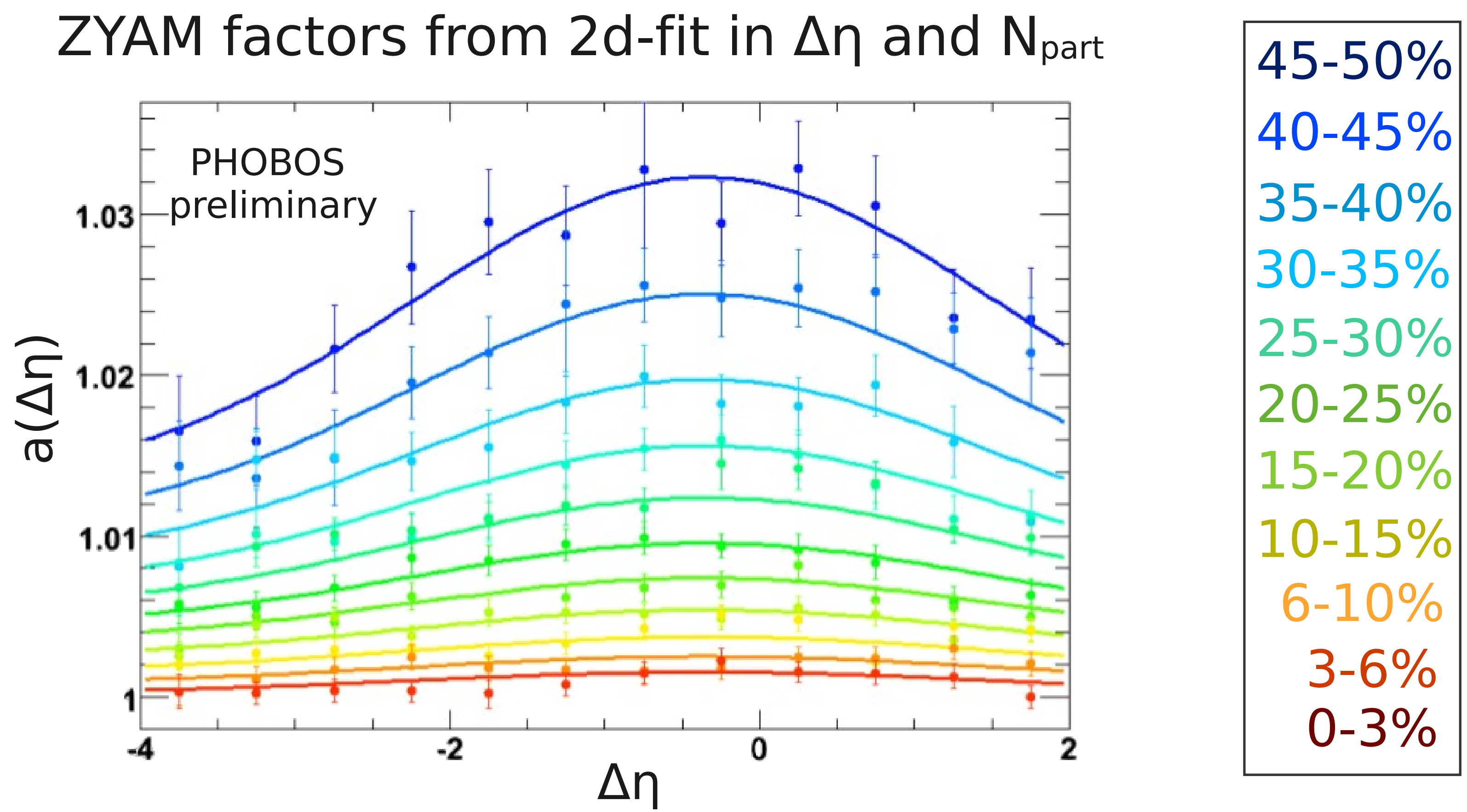



$a(\Delta\eta)\,[1 + 2V(\Delta\eta)\cos(2\Delta\phi)\;]$

PHOBOS, PRL 104 062301 (2010)

# ZYAM implementation

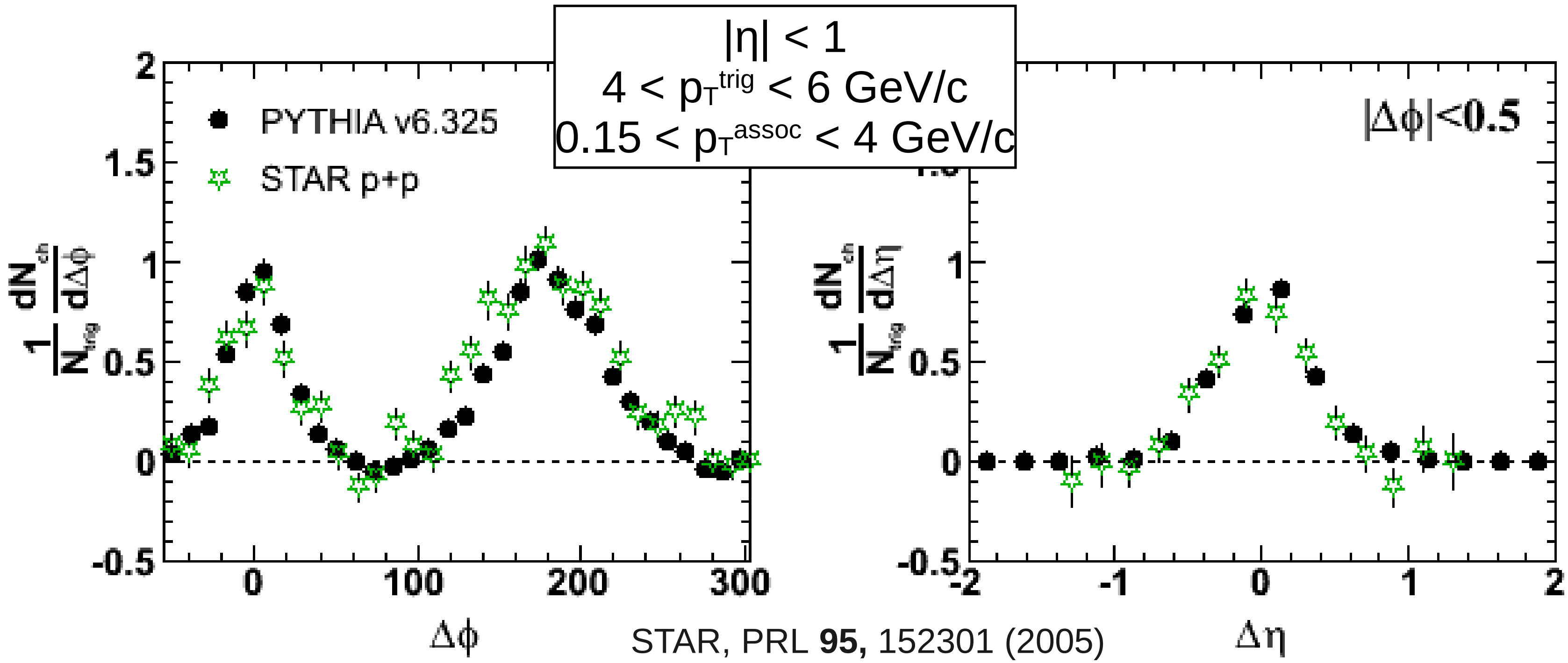





- Constant term: bias of the $p_T$-triggered signal distribution to higher multiplicity
- Gaussian term: Δη correlation structure underneath $v_2$-subtracted Δφ correlations. Width/amplitude/$N_{part}$-dependence same as inclusive correlations

PHOBOS, PRL 104 062301 (2010)

# STAR vs PYTHIA

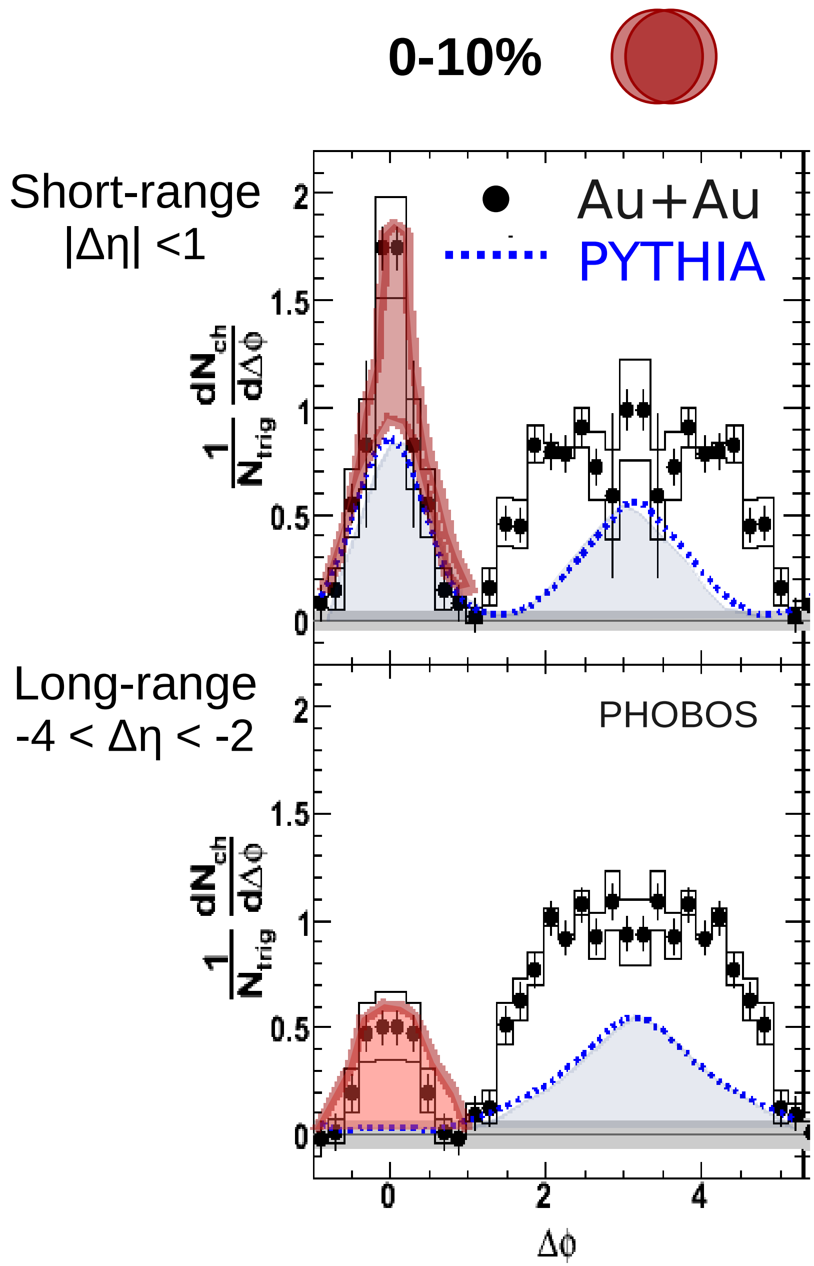



STAR, PRL **95,** 152301 (2005)

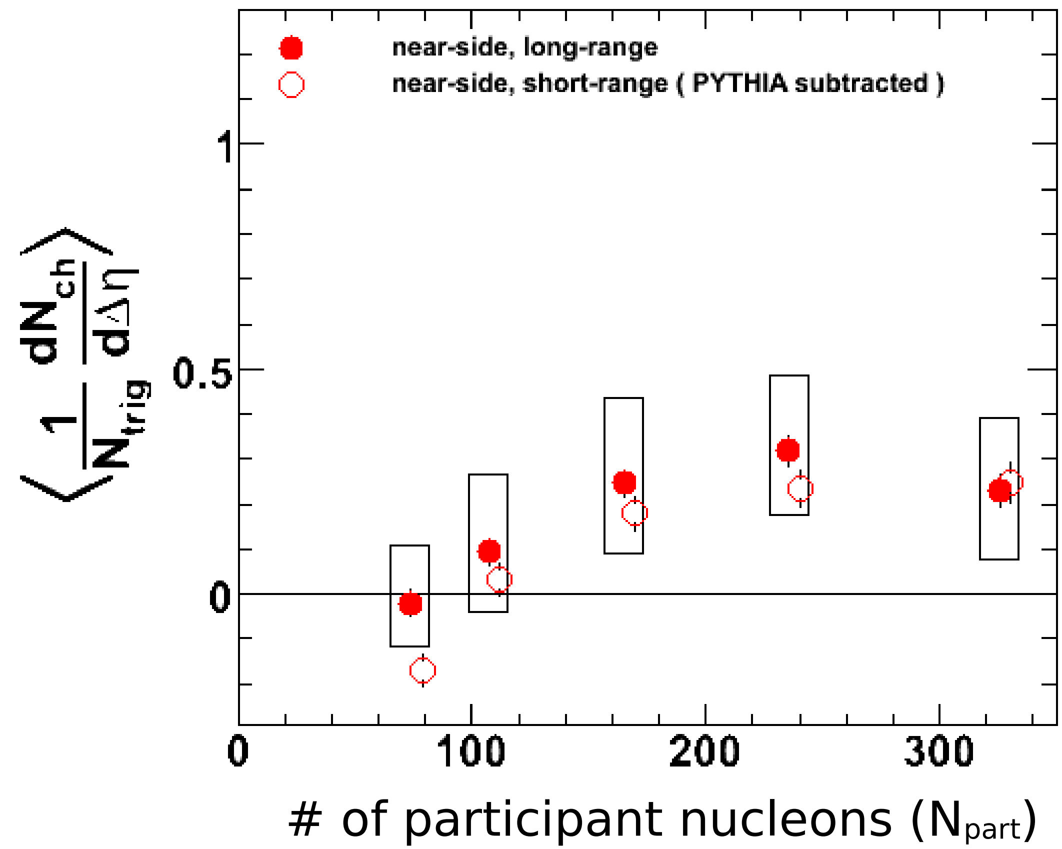
PHOBOS is limited by staticstics in p+p, therefore take PYTHIA as a reference, which matches the STAR measurement well.

# Integrated ridge yield (near side)





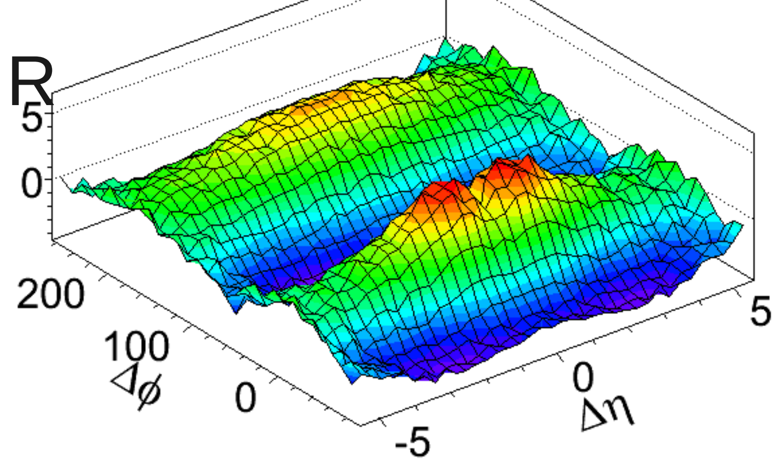

NEAR side
○ short-range minus PYTHIA
● long-range



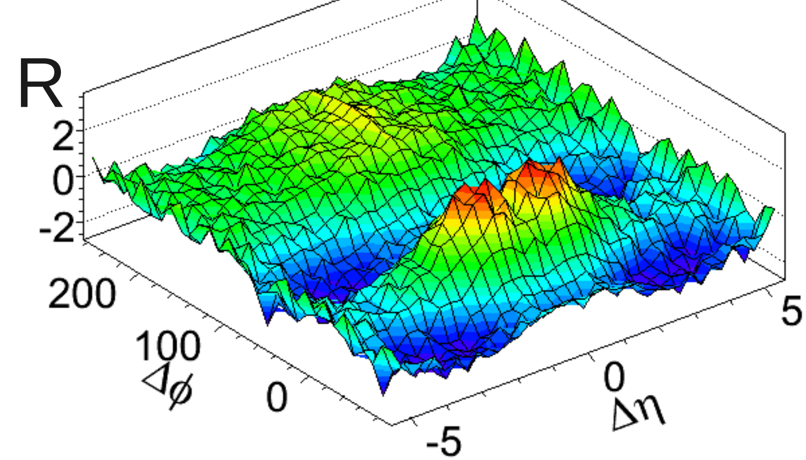

PHOBOS, PRL 104 062301 (2010)

AMPT model: Glauber initial conditions, collective flow

Correlations




Elliptic flow subtracted correlations




AMPT Au+Au 0-20%

AMPT produces similar structures correlation structures


Lin et. al. PRC72, 064901 (2005)
Ma et. Al. PLB641 362 (2006)