Elliptic flow, eccentricity and eccentricity fluctuations

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PHOBOS collaboration

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UNIVERSITY OF MARYLAND

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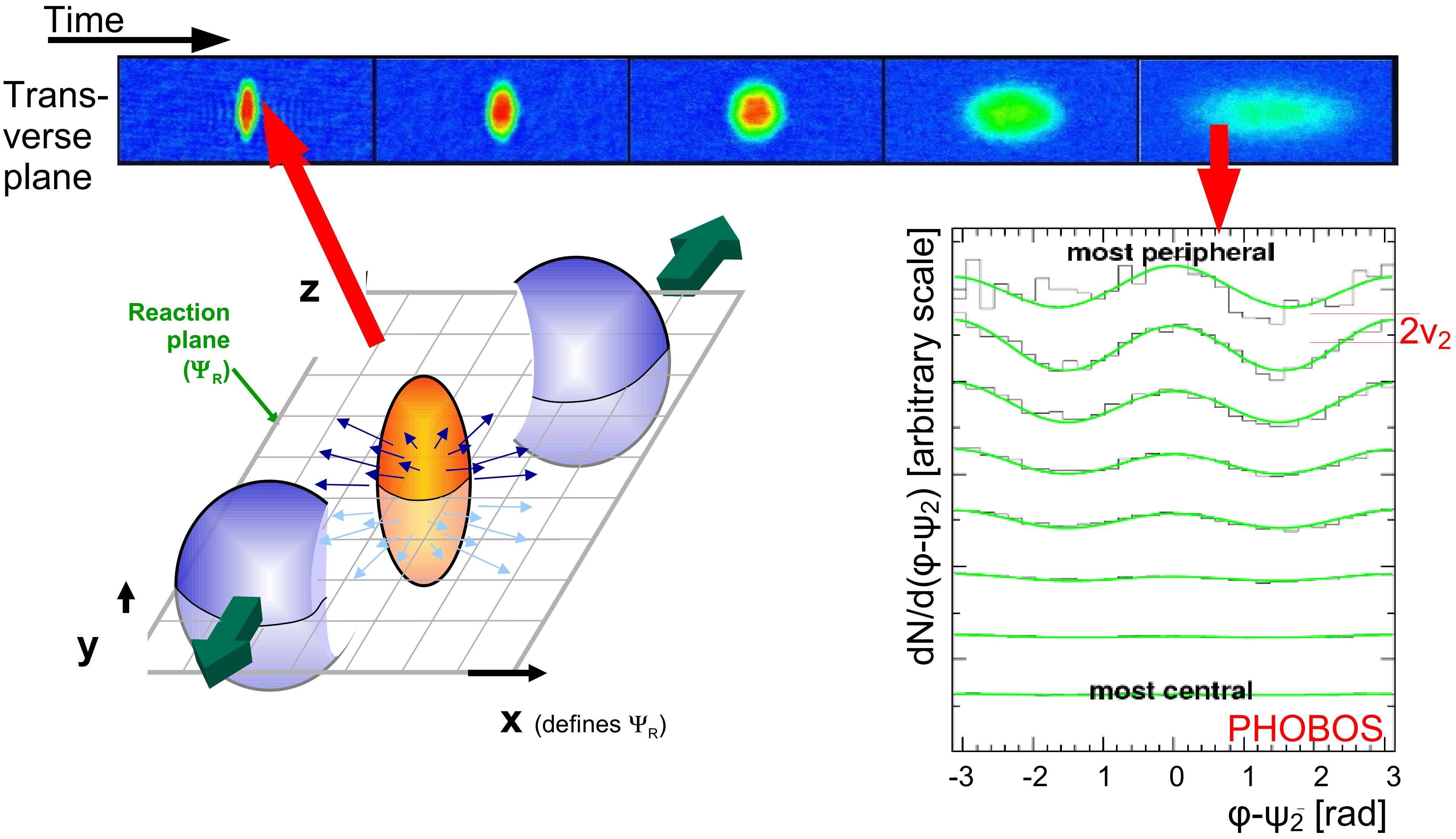
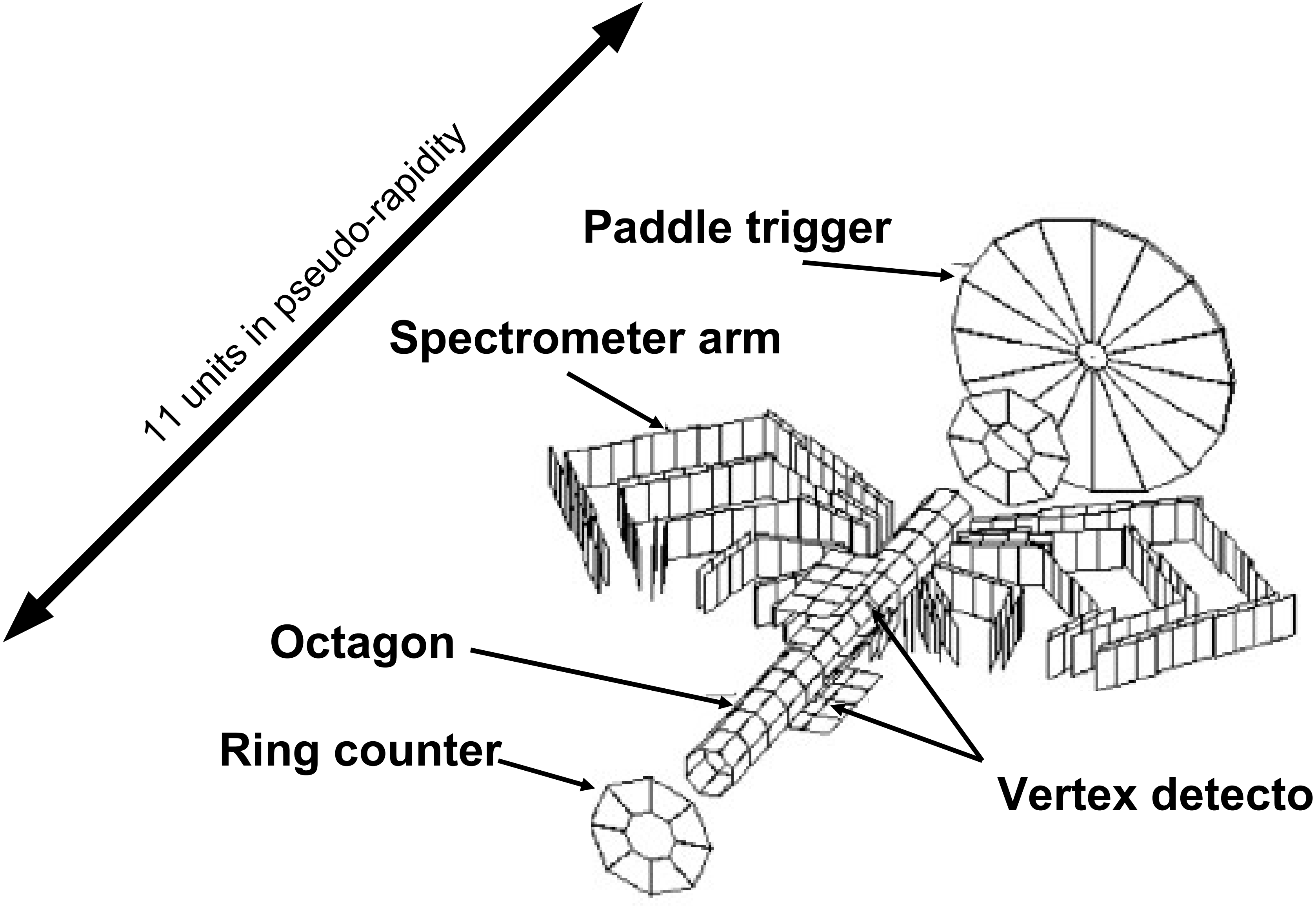
Elliptic flow



Non-central collision:
Initial state eccentricity

Momentum space anisotropy
$v_2 = \langle \cos(2\phi - 2\Psi_R) \rangle$

Flow measurement in PHOBOS



Flow measurement in PHOBOS

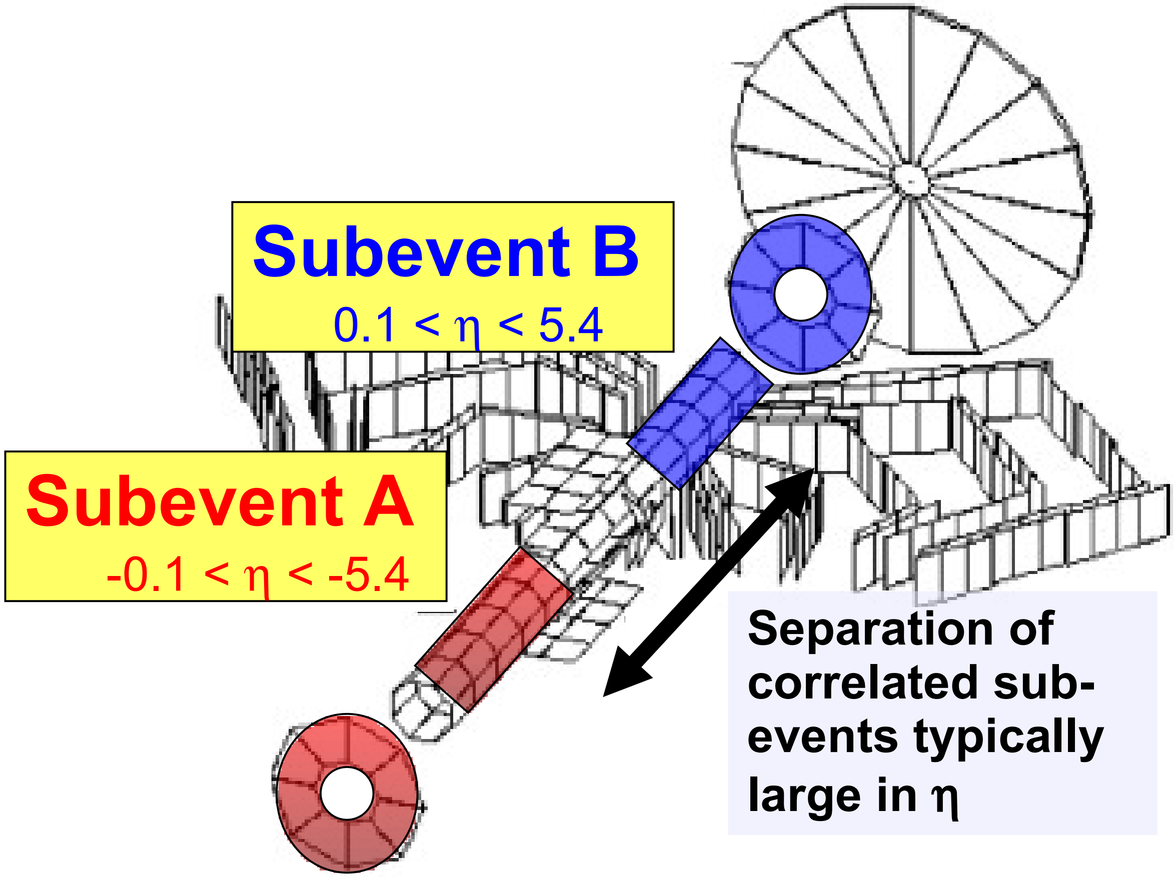
- Reaction-plane / Subevent technique
  - Correlate reaction plane determined from azimuthal pattern of hits in one part of the detector with information from other parts of the detector

$$\tan(2\psi_A) = \frac{\langle \sin(2\phi) \rangle_A}{\langle \cos(2\phi) \rangle_A}$$

$$v_2^{obs} = \langle \cos(2\phi - 2\psi_A) \rangle_B$$

$$v_2 = \frac{\langle v_2^{obs} \rangle_{events}}{\sqrt{\langle \cos(2\psi_A - 2\psi_B) \rangle_{events}}}$$

A.Poskanzer, S.Voloshin, nucl-ex/9805001



Subevent B

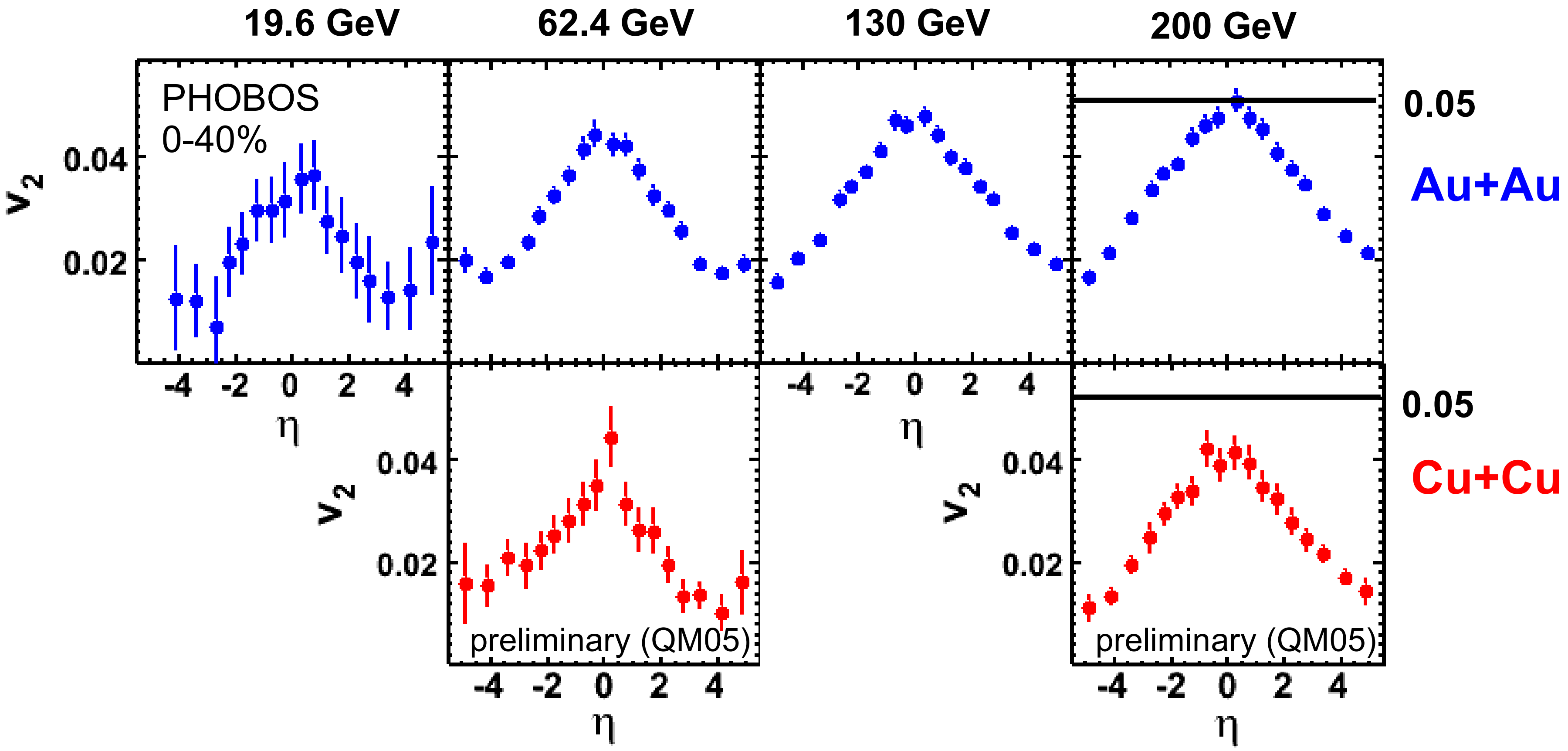
$0.1 < \eta < 5.4$

Subevent A

$-0.1 < \eta < -5.4$

Separation of correlated sub-events typically large in η

Elliptic flow (v_2)



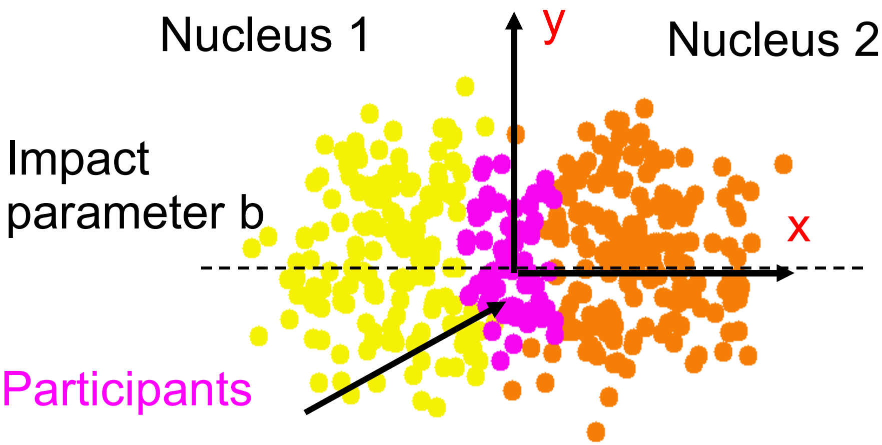
Cu+Cu about 20% lower than Au+Au

Au+Au: PRL 94 122303 (2005)
Cu+Cu: nucl-ex/0510031 (preliminary)

Relating system size and eccentricity

- Glauber Monte Carlo
  - Radial distribution of nucleons (in nucleus) drawn from Wood-Saxon distribution
  - Isotropic angular distribution
  - Separate by impact parameter
  - Nucleons travel on straight-line paths and interact inelastically when

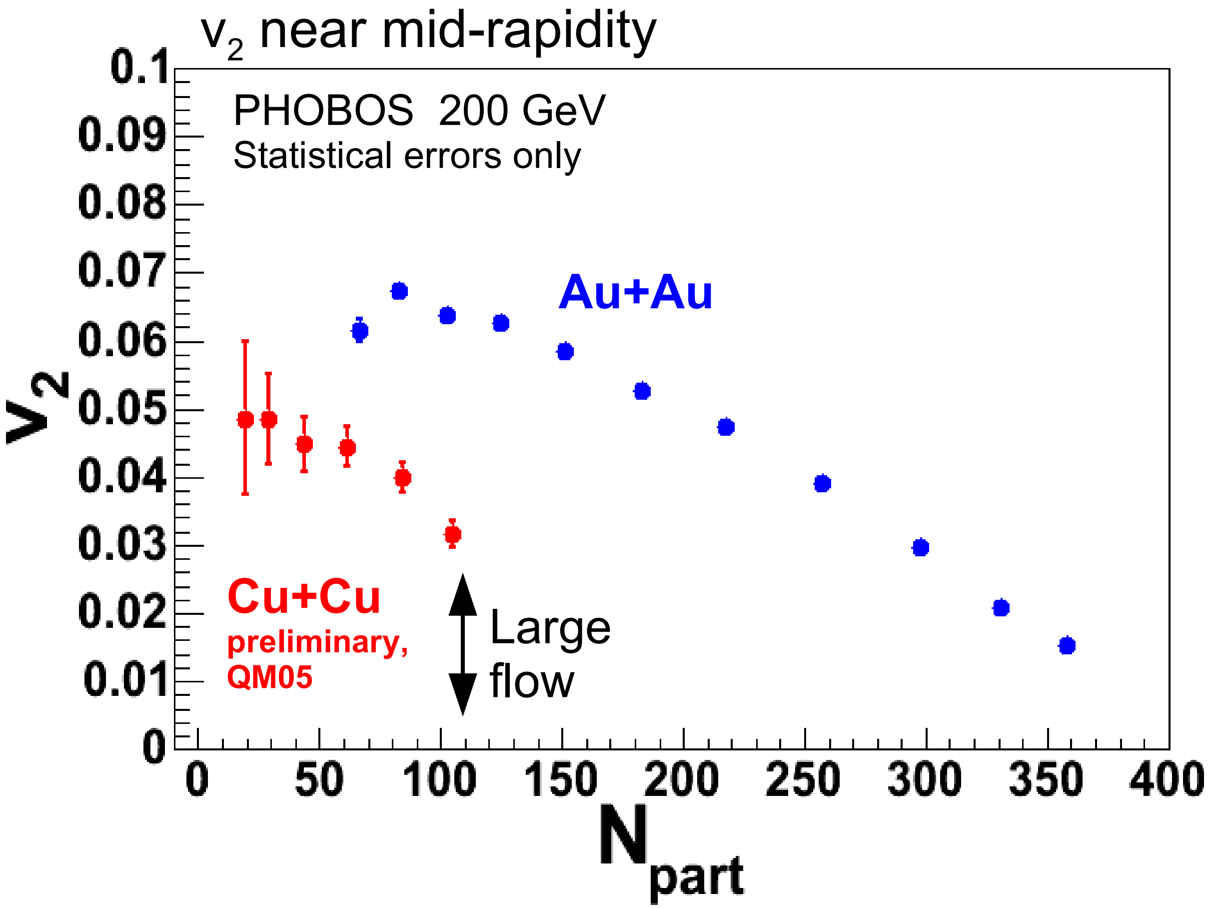
$$d=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}<\sqrt{\sigma_{NN}/\pi}$$

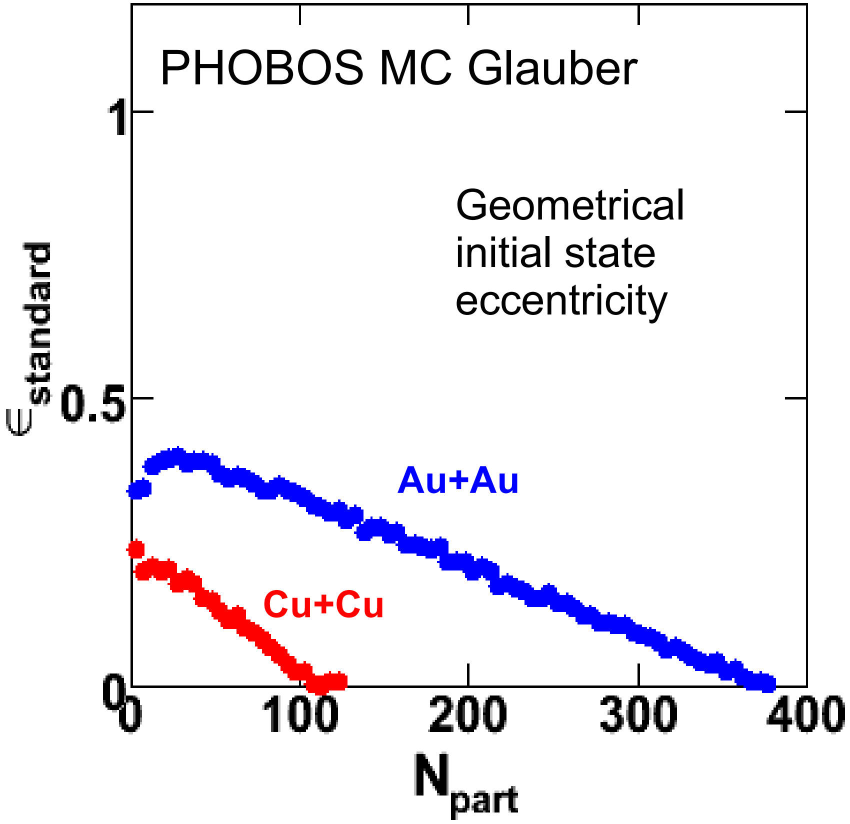


- Centrality of collision
  - #Participants
    - Nucleons that interact at least once
  - Related to cross section and impact parameter range
- Eccentricity of collision zone
  - Given by participants position distributions

Eccentricity: $\epsilon_{std}=\frac{\sigma_y^2-\sigma_x^2}{\sigma_y^2+\sigma_x^2}$

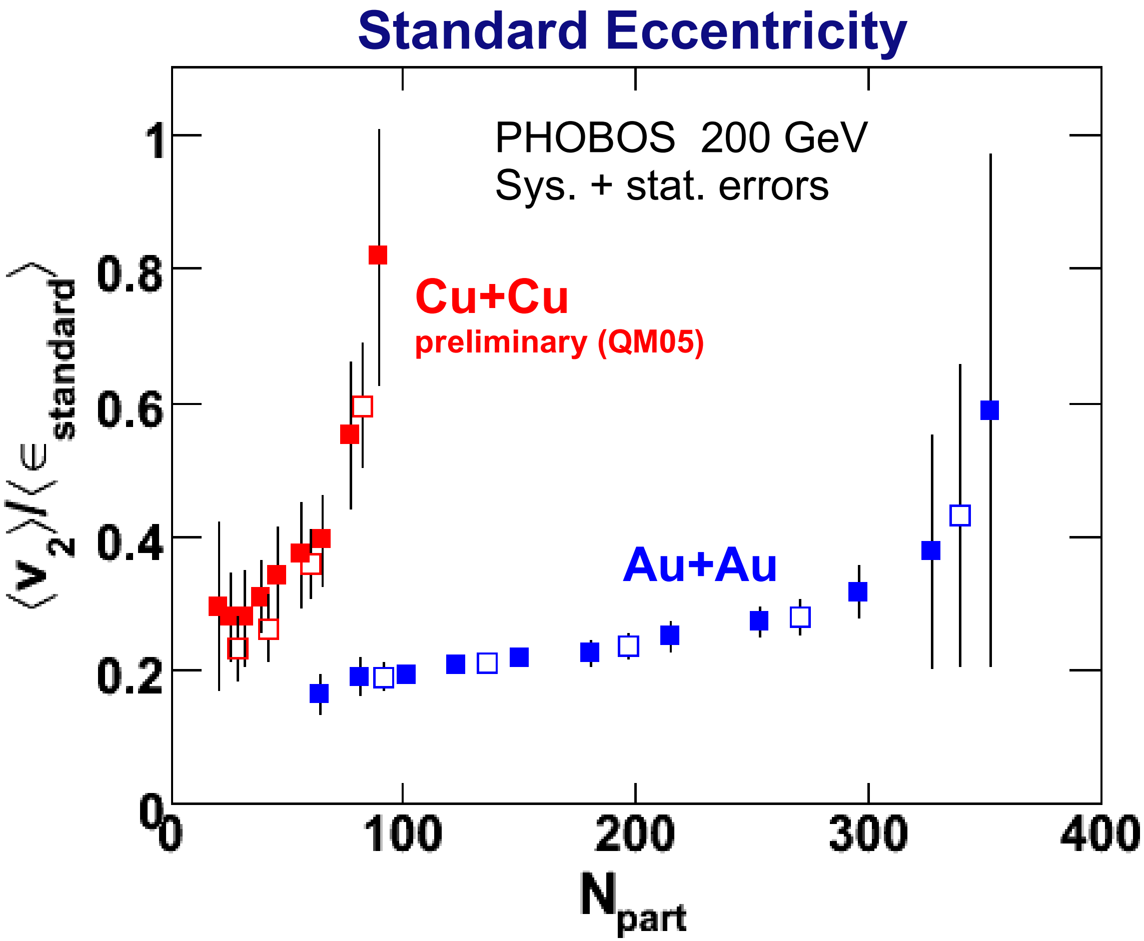
Elliptic flow in Cu+Cu and Au+Au





Au+Au: PRC 72, 051901 (2005)
Cu+Cu: prel. QM05, nucl-ex/0510042

Scaled elliptic flow vs N_{part}

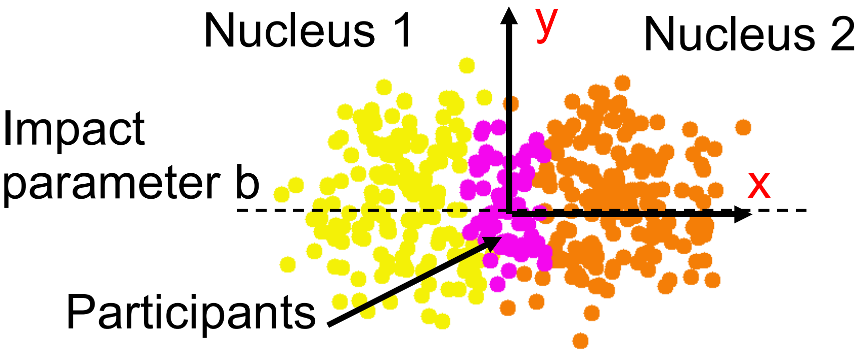


No agreement between Cu+Cu and Au+Au scaled by $\epsilon_{standard}$

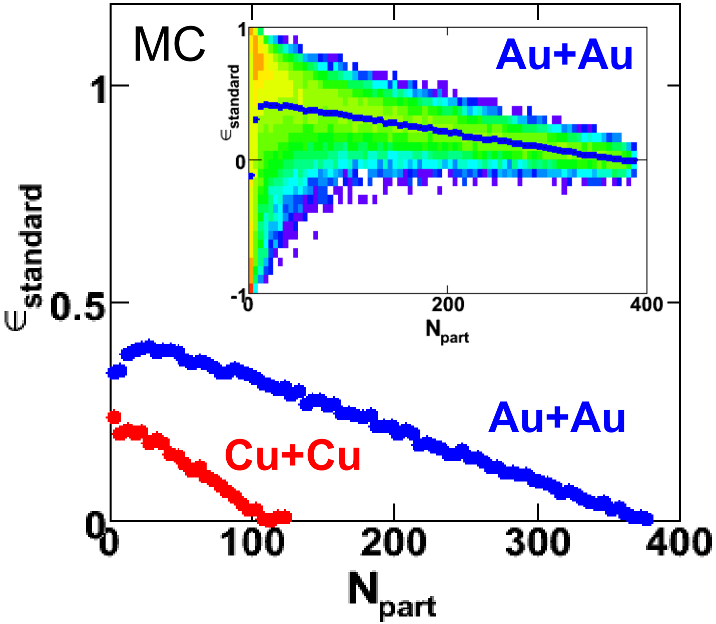
Au+Au: PRC 72, 051901 (2005)
Cu+Cu: prel. QM05, nucl-ex/0510042

Standard eccentricity calculation

Standard Eccentricity

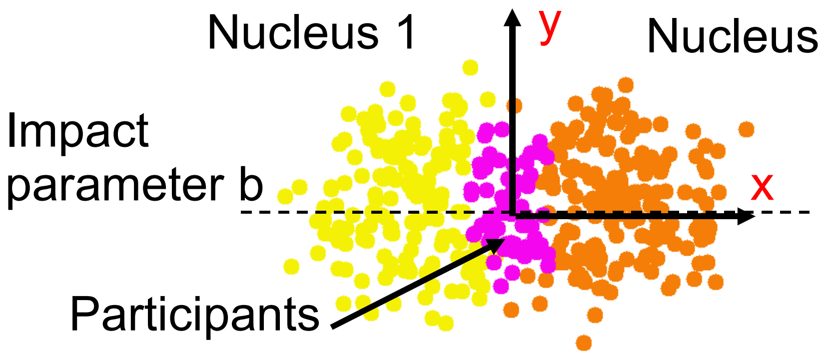


$$\epsilon_{std} = \frac{\sigma_y^2 - \sigma_x^2}{\sigma_y^2 + \sigma_x^2}$$

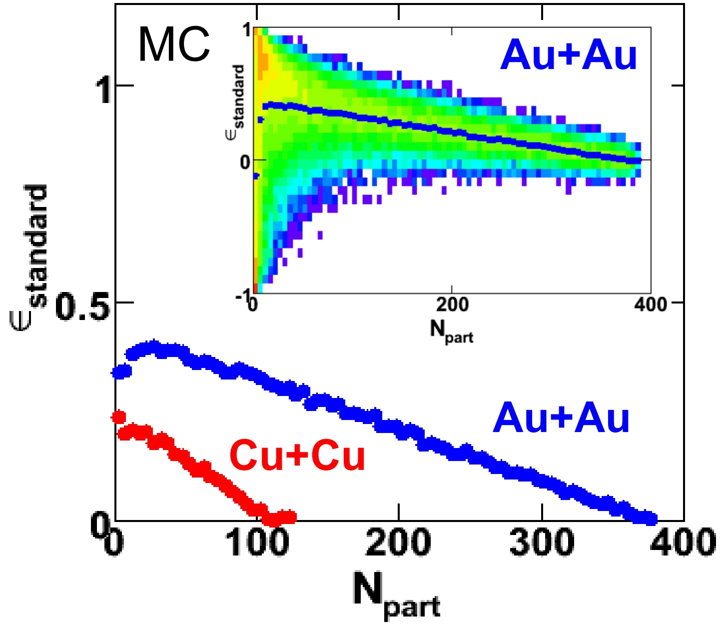


Participant eccentricity calculation

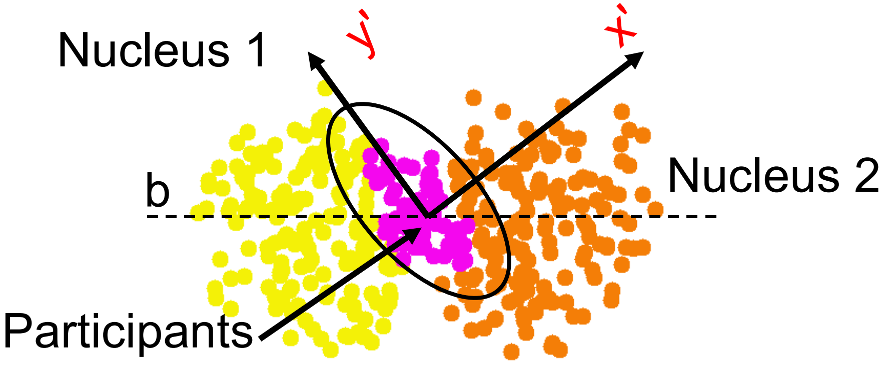
Standard Eccentricity



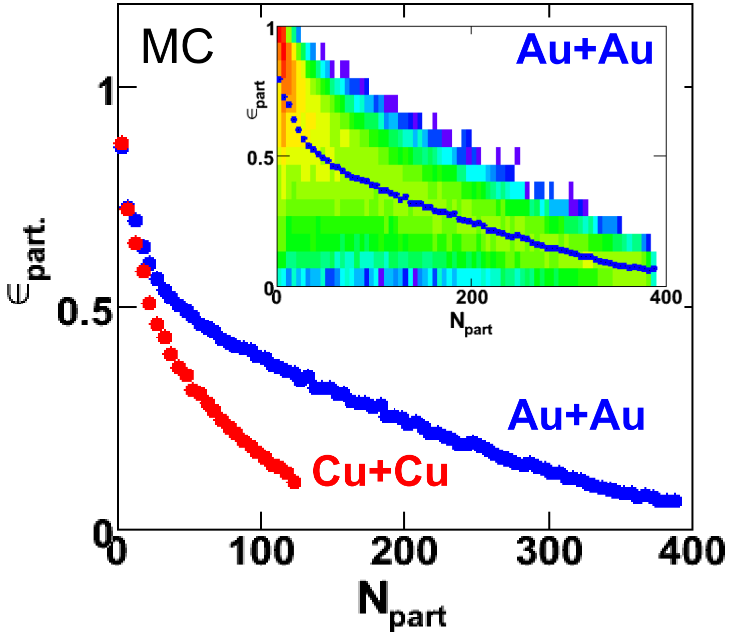
$$\epsilon_{std} = \frac{\sigma_y^2 - \sigma_x^2}{\sigma_y^2 + \sigma_x^2}$$



Participant Eccentricity



$$\epsilon_{part} = \frac{\sqrt{(\sigma_y^2 - \sigma_x^2)^2 - 4\sigma_{xy}^2}}{\sigma_y^2 + \sigma_x^2}$$

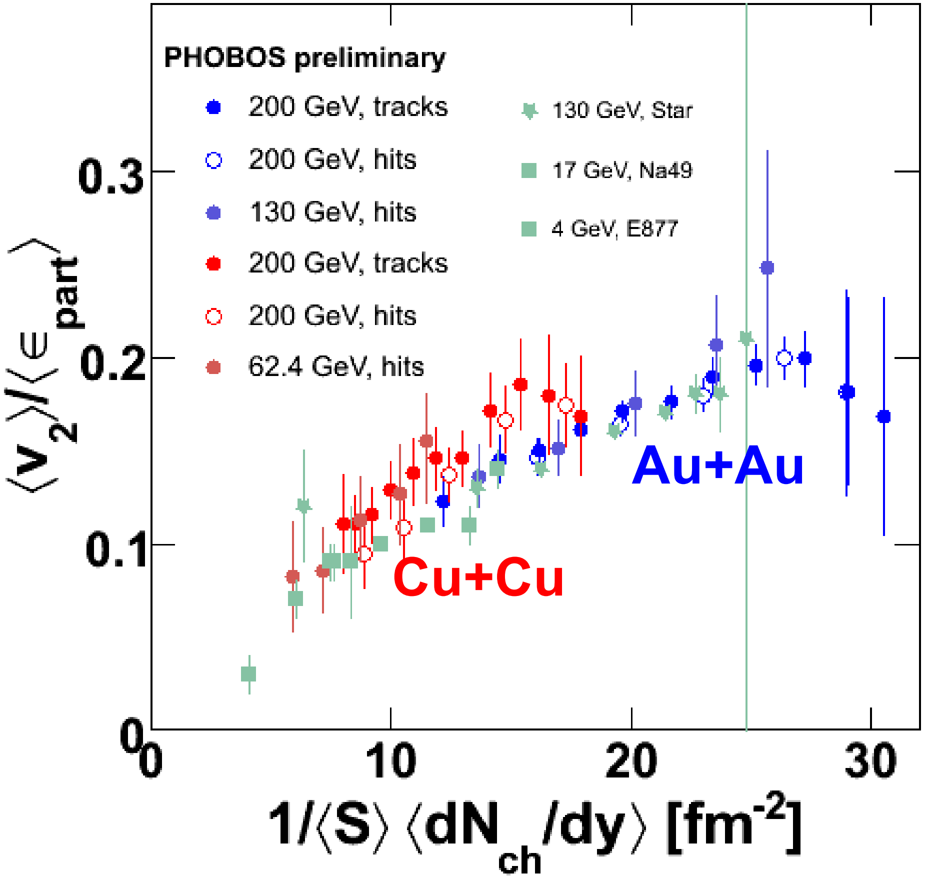


Participant eccentricity scaling

- Caution: We used $\epsilon_{participant}$ for PHOBOS data. Important for Cu-Cu, less critical for Au-Au.
- Scale $v_2(\eta)$ to $\sim v_2(y)$ (10% lower)
- Scale $dN/d\eta$ to be $\sim dN/dy$ (15% higher)
- STAR and AGS Au+Au and CERN Pb+Pb results are not scaled by $\epsilon_{participant}$
- S is overlap area (MC Glauber)

STAR, PRC 66 034904 (2002)
Voloshin, Poskanzer, PLB 474 27 (2000)
Heiselberg, Levy, PRC 59 2716, (1999)
Au+Au: PRC 72, 051901 (2005)
Cu+Cu: prel. QM05, nucl-ex/0510042

Participant Eccentricity

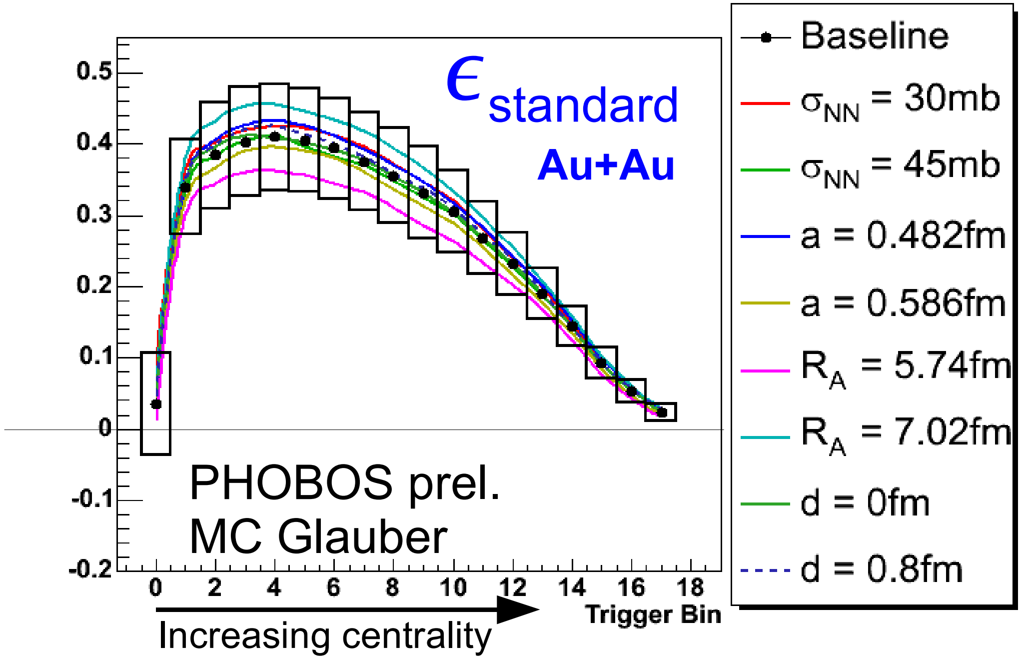


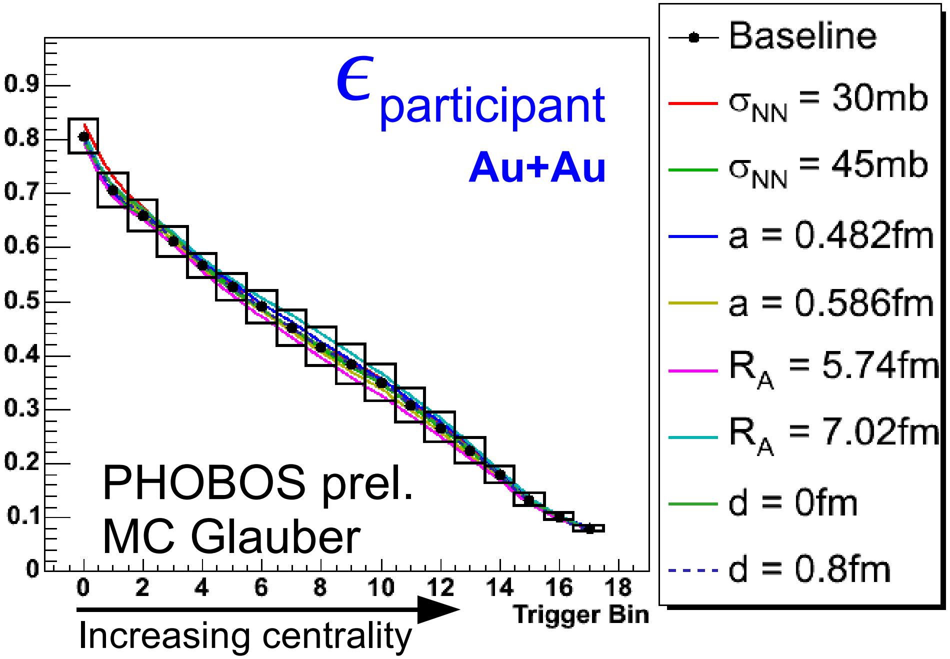
Approximate scaling between Cu+Cu and Au+Au

Since QM 2005

- Examine properties of participant eccentricity
  - Study robustness wrt to geometry parameters
  - Connection to v_2 measurements and hydro
    - Work in collaboration with Uli Heinz
- Measure higher moments in v_2

Robustness with geometry variables





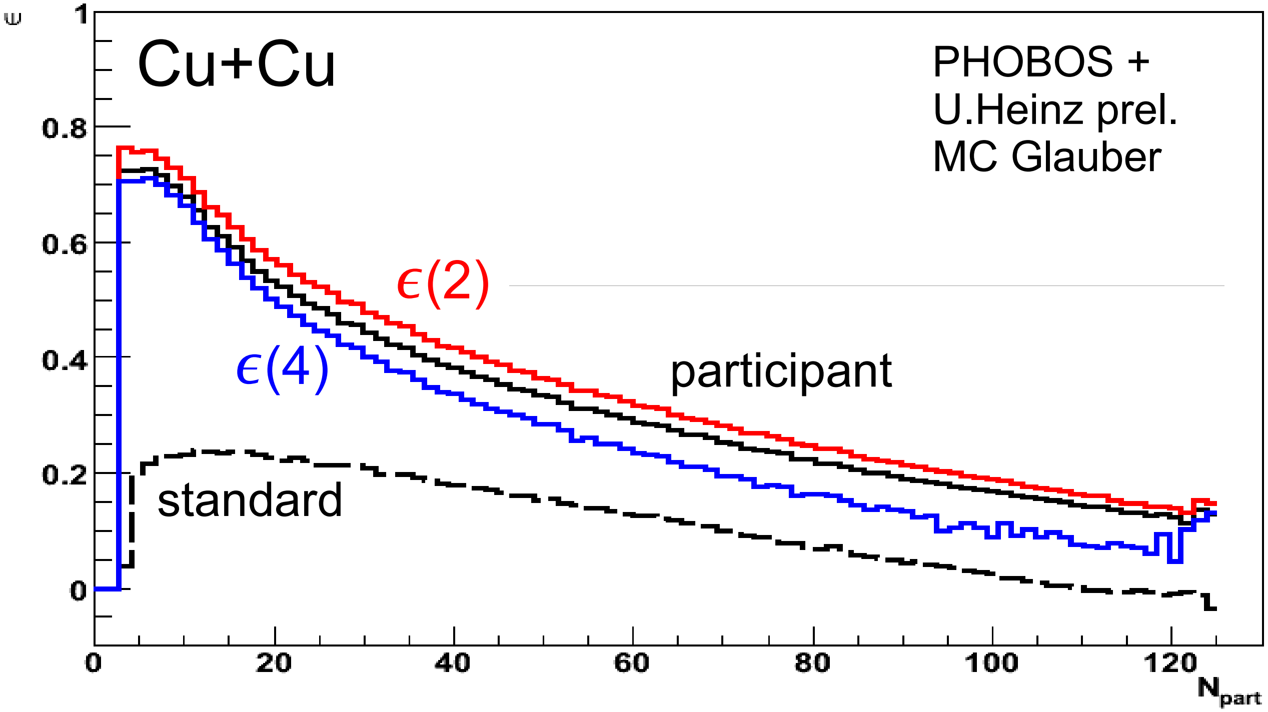
- Variation of
  - Nucleon-nucleon cross section (30-45mb)
  - Nuclear radius (±10% from the nominal value)
  - Skin depth (0.482-0.586fm)
  - Minimum separation distance between nucleons (d=0-0.8fm)

$$\rho(r) = \frac{\rho_0}{1+\exp((r-R)/a)}$$

$\epsilon_{\text{participant}}$ even slightly more robust than $\epsilon_{\text{standard}}$

Moments of eccentricity

- If one measures v_2 with two-particle correlations
  $v_2(2)=\sqrt{\langle\cos 2(\phi_1-\phi_2)\rangle}=\sqrt{\langle v_2^2\rangle}$
- If v_2 fluctuates prop. to ϵ_{part}
  - Scaling with $\epsilon(2)=\sqrt{\langle\epsilon_{part}^2\rangle}$
- For 4-cumulant method $v(4)=[2\langle v^2\rangle^2-\langle v^4\rangle]^{1/4}$
  - Scaling with $\epsilon(4)=[2\langle\epsilon^2\rangle^2-\langle\epsilon^4\rangle]^{1/4}$



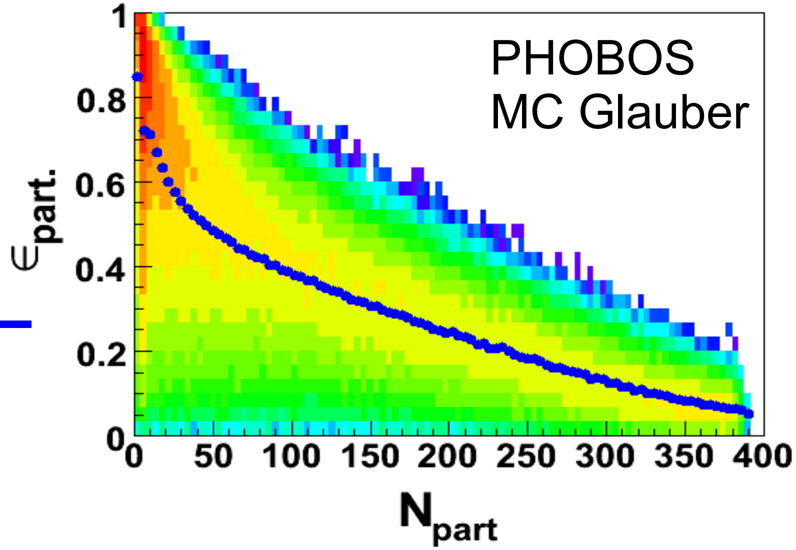
$\epsilon\{2\}$ rather similar to ϵ_{part}, unlike $\epsilon\{4\}$ that is between ϵ_{std} and ϵ_{part}

M.Miller, R.Snellings, nucl-ex/0312008
R.Bhalerao, J.Ollitrault, nucl-th/0607009

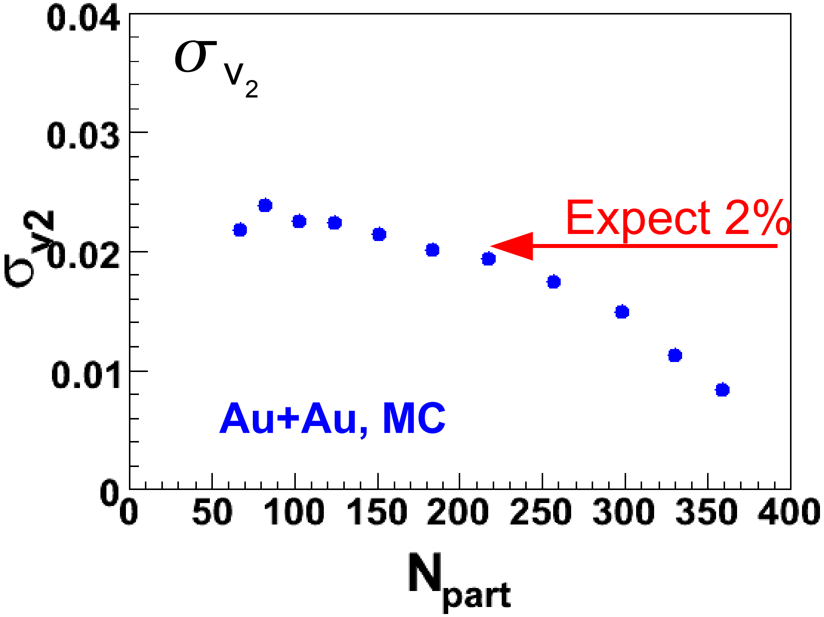
Expected elliptic flow fluctuations

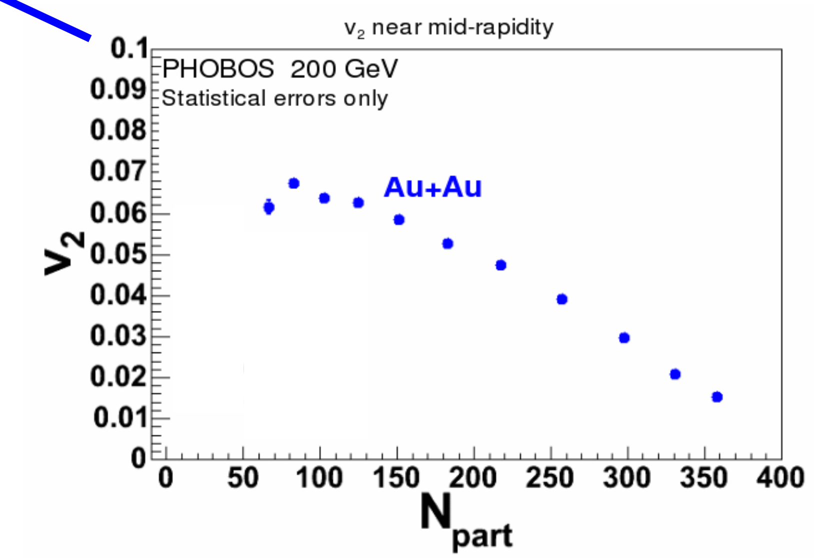
Assuming $v_2 \propto \epsilon_{part}$, the participant eccentricity model predicts

$$\sigma_{v_2} = \frac{\sigma_{\epsilon_{part}}}{\epsilon_{part}} v_2$$



Expected σ_{v2} from fluctuations in ε_{part}





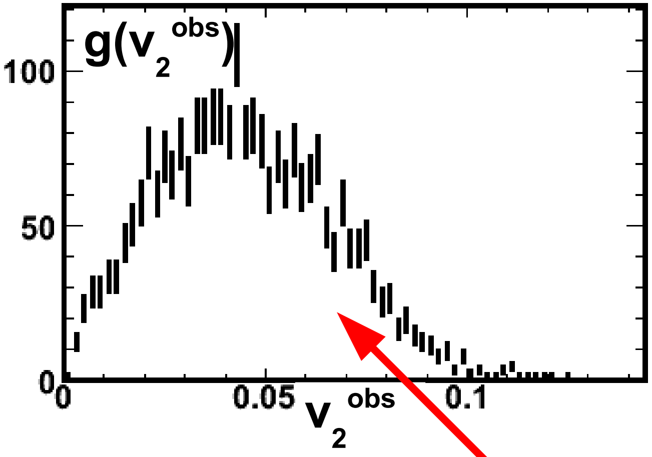
Measuring elliptic flow fluctuations

- Ongoing analysis on 200GeV Au+Au
  - NO DATA will be shown
  - Overview of the measurement
    - Studies on fully simulated MC events
  - Details are in nucl-ex/0608025

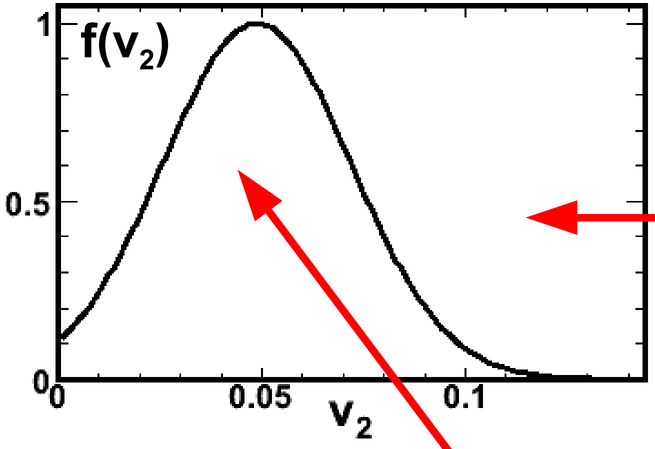
B.Alver et.al. (PHOBOS), nucl-ex/0608025

Measuring elliptic flow fluctuations

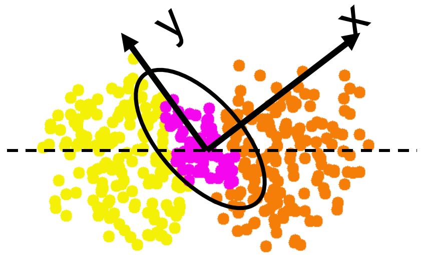
Observed v2 distribution



True v2 distribution

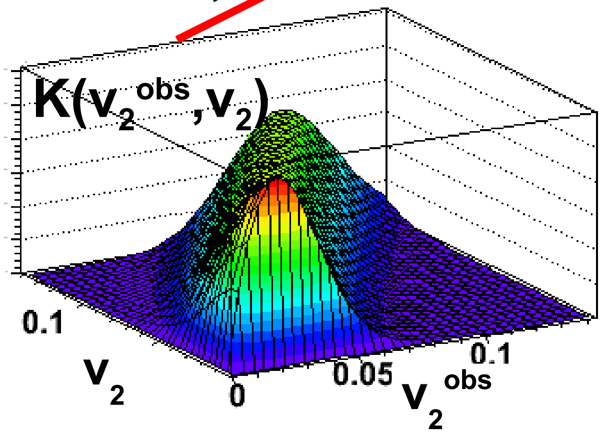


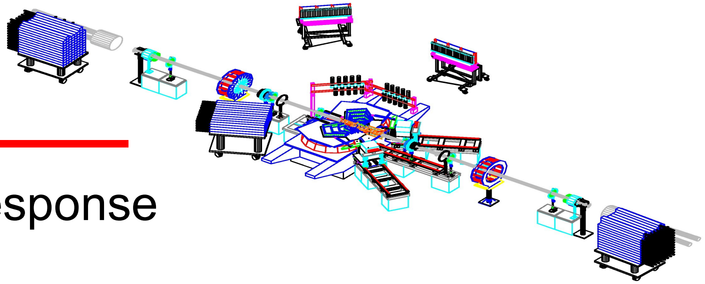
Participant eccentricity



$$g(v_2^{obs}) = \int_0^\infty K(v_2^{obs}, v_2)\, f(v_2)\, dv_2$$

Kernel

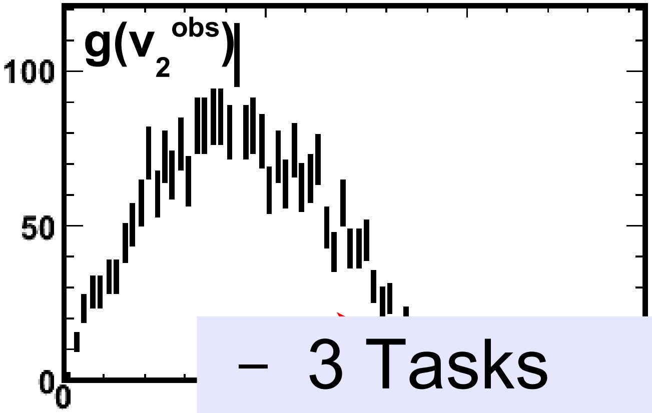




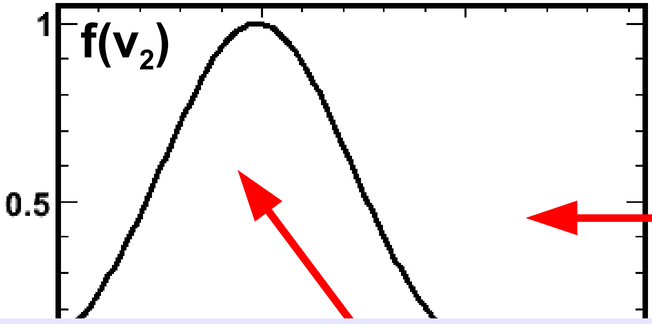
Detector response

Measuring elliptic flow fluctuations

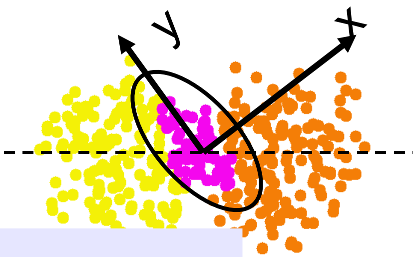
Observed v2 distribution



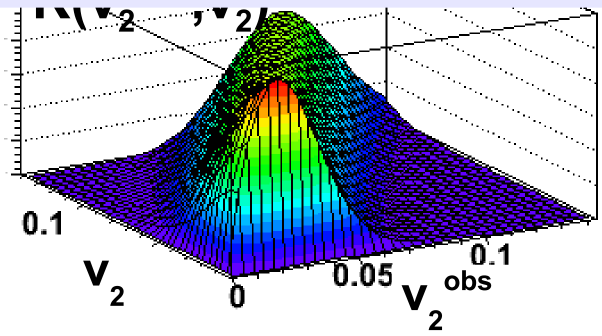
True v2 distribution

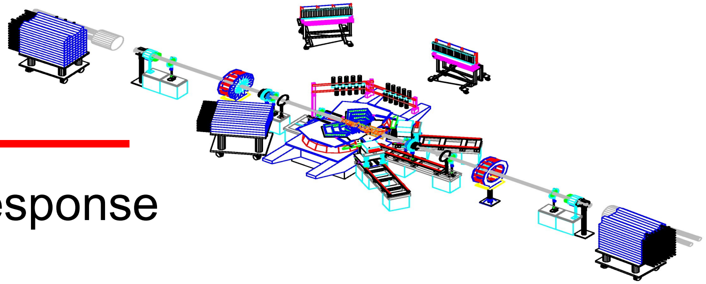


Participant eccentricity



- 3 Tasks
  - Measurement of v_2^{obs} event-by-event: $g(v_2^{obs})$
  - Construction of the kernel: $K(v_2^{obs},v_2)$
  - Extraction of dynamical fluctuations: $f(v_2)$





Detector response

Event-by-event measurement of v_2^{obs}

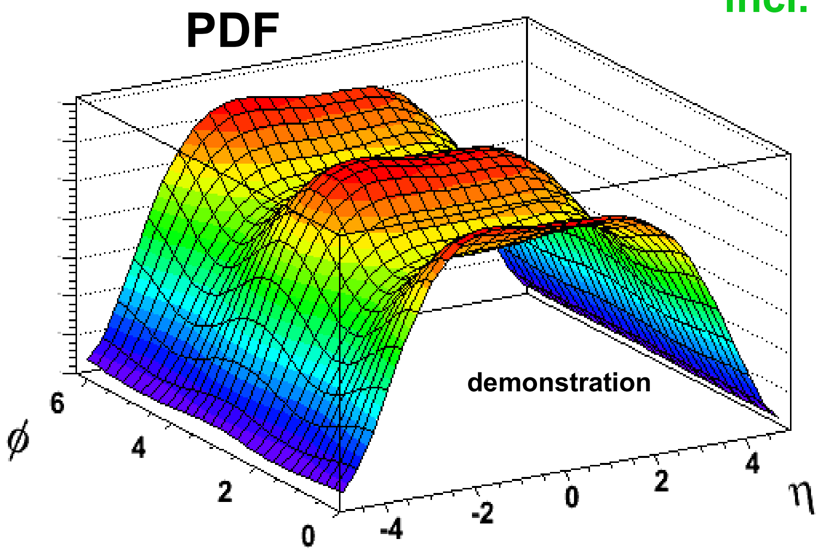
- Probability Distribution Function (PDF) for hit positions:

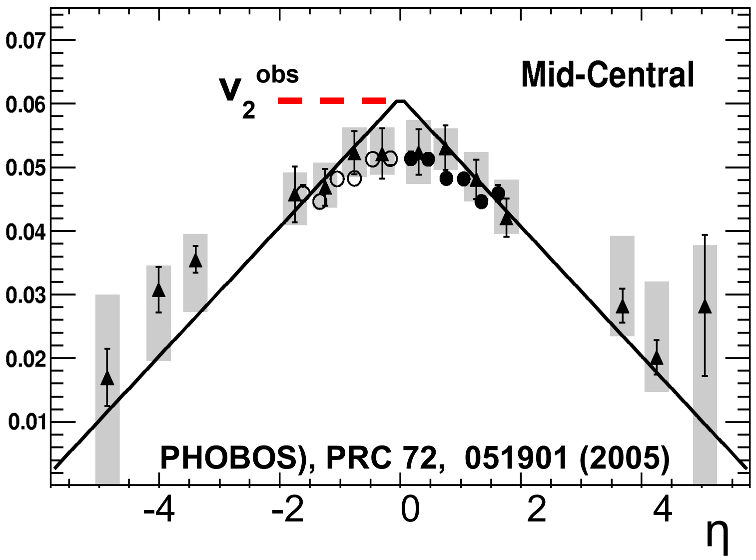
$$P(\eta,\phi;\, v_2^{obs},\phi_0)=\underbrace{p(\eta)}_{\uparrow}\underbrace{[1+2v_2(\eta)\cos(2\phi-2\phi_0)]}_{\uparrow}$$

Normalization incl. acceptance

Probability of hit in (φ,η)

$$v_2(\eta)=v_2^{obs}\left(1-\frac{|\eta|}{6}\right)$$





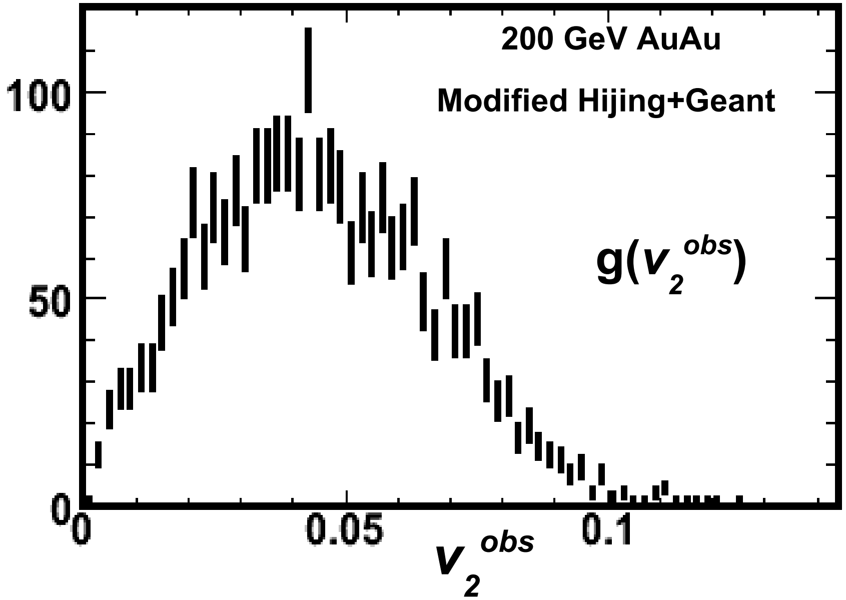
PHOBOS), PRC 72, 051901 (2005)

- Maximize the likelihood function

$$L(v_2^{obs},\phi_0)=\prod_{i=1}^{n} P(\eta_i,\phi_i;\, v_2^{obs},\phi_0)$$

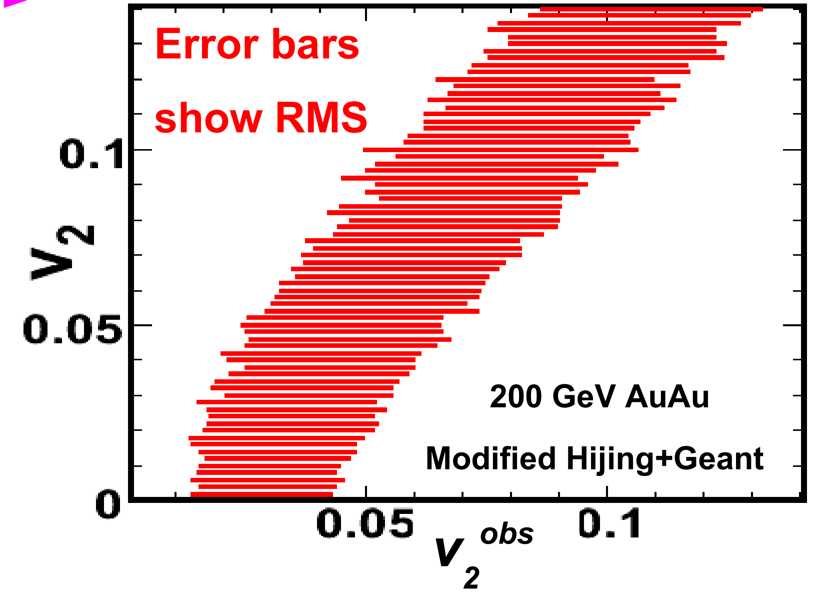
Determining the kernel

Observed v_2^{obs} distribution in a sample



MC

Mean and RMS of v_2^{obs} in slices of v_2



- Construction of kernel: "Measure" v_2^{obs} distribution in bins of v_2 in MC
- 2 small complications
  - Kernel really depends on multiplicity (centrality): $K(v_2^{obs}, v_2, n)$
    - n = number of hits on the detector
    - Determine v_2^{obs} distribution in bins of v_2 and n.
  - Deal with low statistics in bins by determining smooth functions

Extracting dynamical fluctuations

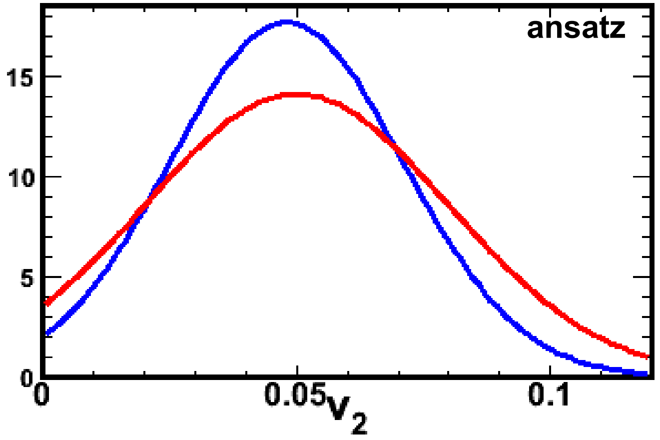
$$\underline{g(v_2^{obs})} = \int_0^\infty \underline{K(v_2^{obs}, v_2)}\, \underline{f(v_2)}\, dv_2$$

known

Gaussian Ansatz:

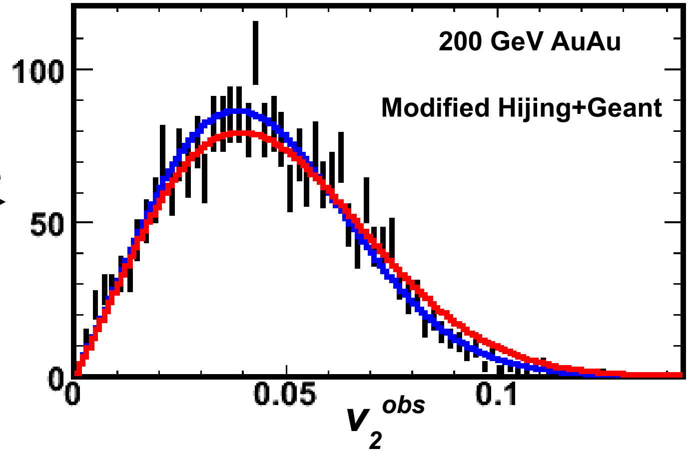
$$f(v_2) = \exp\left[\frac{-(v_2 - \langle v_2 \rangle)^2}{2\sigma_{v_2}^2}\right]$$

Trials for f(v_2)



Use kernel + integrate →

Comparison with sample



Compare expected g(v_2^{obs}) for Ansatz with measurement

Minimum $\chi^2 \rightarrow \langle v_2 \rangle$ and σ_{v2}

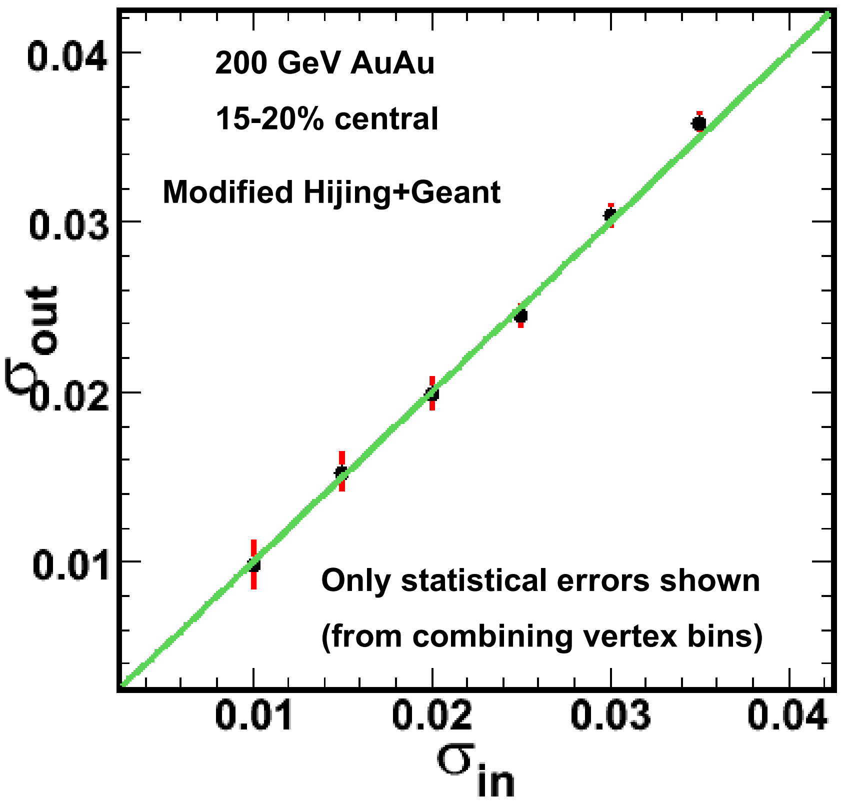
Verification of analysis method

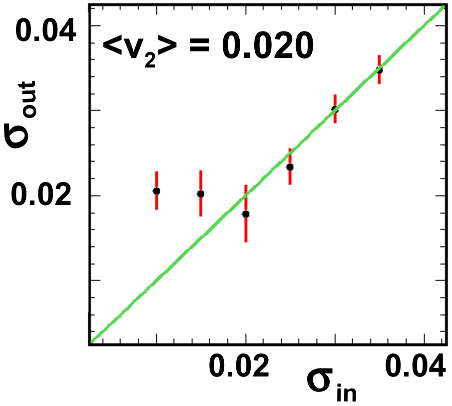
- Run analysis on Modified Hijing
  - Shape $v_2(\eta) = v_2(0) \cdot (1-|\eta|/6)$
    - Same as the assumption in our pdf
  - Analysis done in 10 collision vertex bins
    - Final results are averaged
- Kernel
  - 0-40% central events used to construct the kernel
- Input sample
  - 15-20% central events used as sample
  - $v_2(0)$ given by a Gaussian distribution
    - Same as our Ansatz

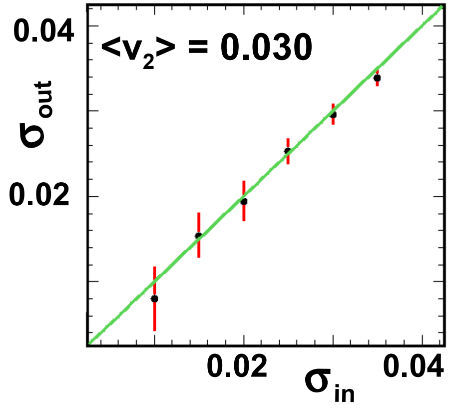
Verification of analysis method

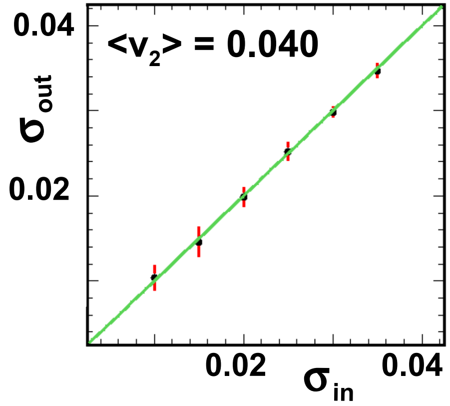
- Run analysis on Modified Hijing
  - Input fluctuations are successfully reconstructed

$\langle v_2 \rangle = 0.050$



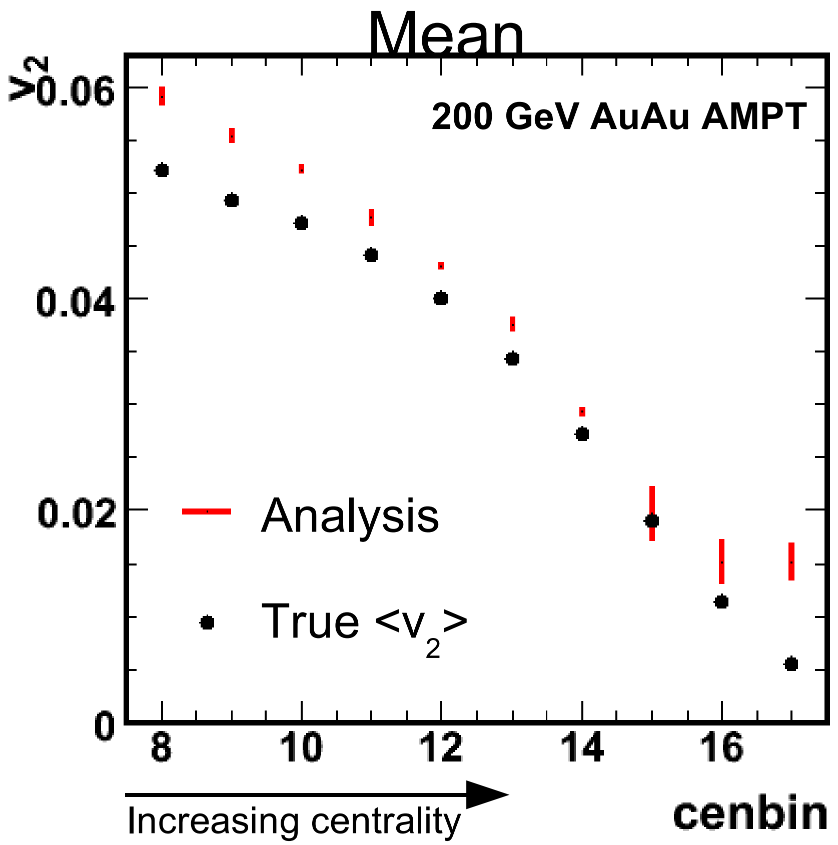


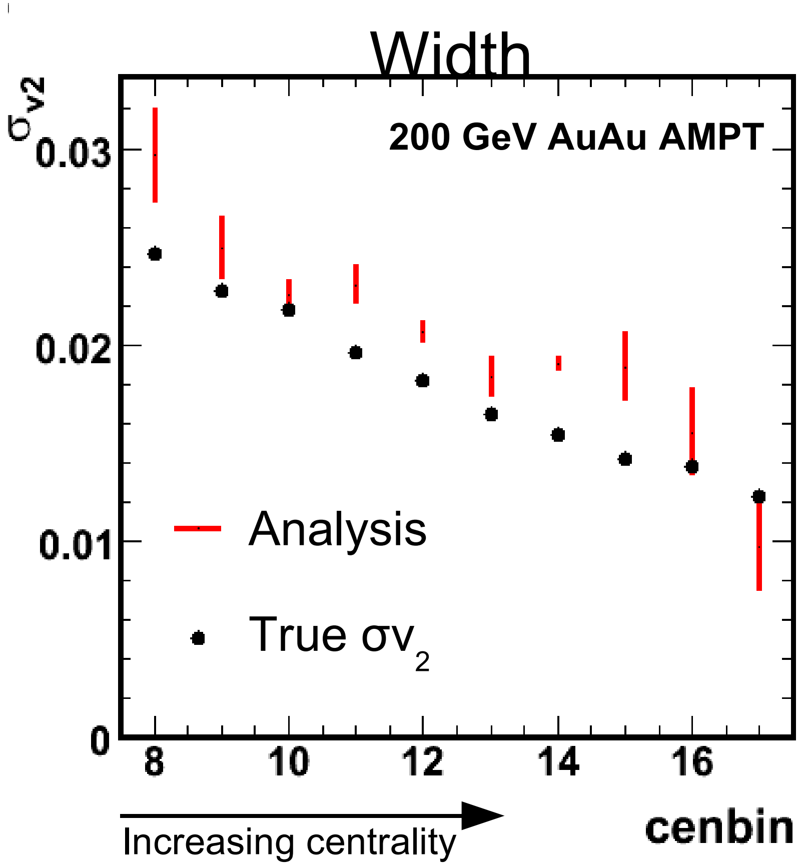




Systematic verification with AMPT

- Run analysis on AMPT
  - Kernel is constructed from Hijing as before
  - Input fluctuations are successfully reconstructed



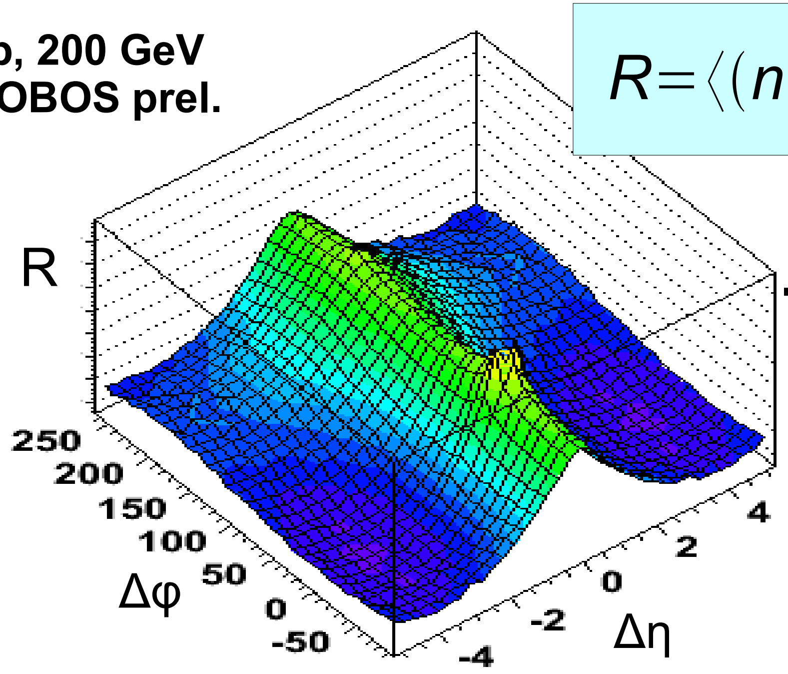


Non-flow contributions to flow fluctuations

- Non-flow correlations mimic dynamical fluctuations and will contribute to the width of the v_2 distribution
  - The resolution of our method depends on the kernel
    - Modified Hijing: particle multiplicity defines the resolution
    - Data (AMPT): clusters flow and therefore the cluster multiplicity determines the resolution
  - The fluctuations we measure are real (present at particle level) but might not be the ones we are after
- Kernel could compensate for non-flow effects if they are correctly described by the MC used to construct it
  - Construct and tune MC on data
  - Two-particle correlation measurements can be used as input to disentangle the different contributions

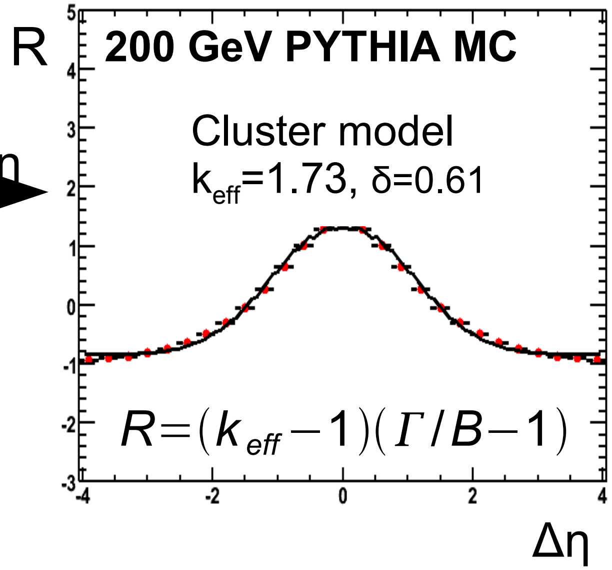
Two-particle angular correlations in p+p

p+p, 200 GeV
PHOBOS prel.



$$R = \langle (n-1)(\frac{F_n}{B_n} - 1) \rangle$$

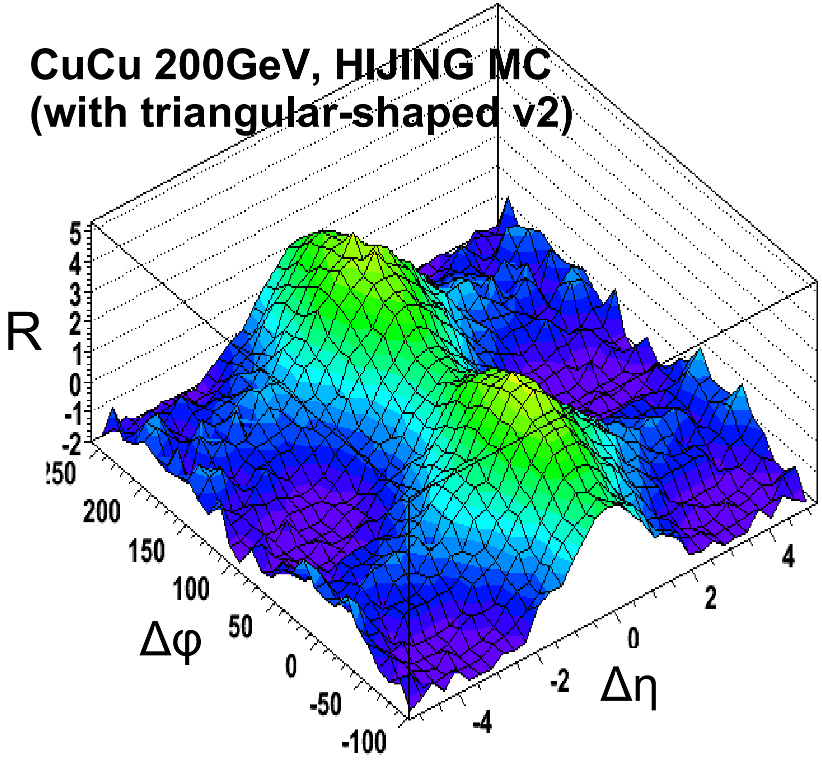
Projection onto Δη
(altern. onto Δφ)



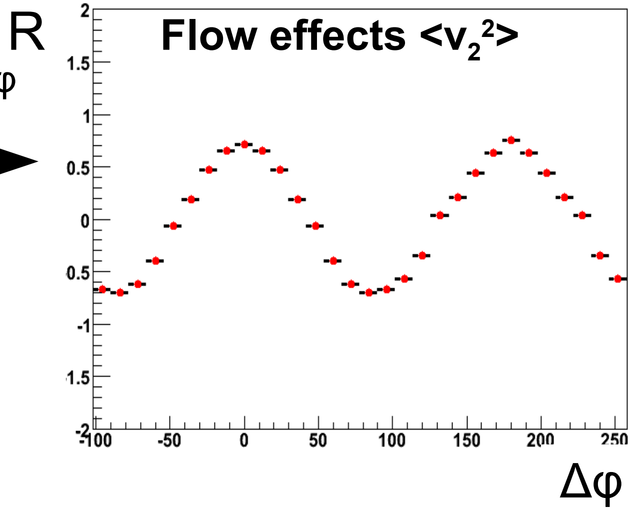
- Construction of R, event-by-event, weighted by event multiplicity
  - Full φ and large |η|≤3 coverage (|η|≤5 for future studies)
  - Single hit in silicon layer instead of particle information
    - Need special care for secondary contamination
  - Study soft physics (No trigger particle)
    - Clusters

Will be published soon

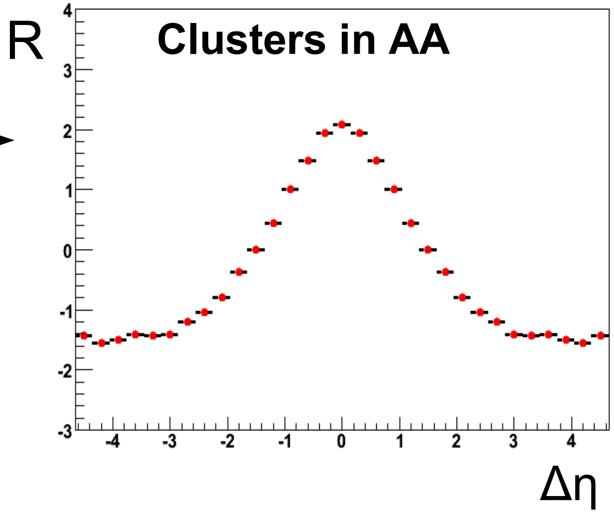
Two-particle angular correlations in A+A


CuCu 200GeV, HIJING MC (with triangular-shaped v2)

Projecting onto Δφ from 0<|Δφ|<180


Flow effects $\langle v_2^2 \rangle$

Projecting onto Δη from 0<|Δη|<6


Clusters in AA

Comprehensive study of two-particle correlations in p+p, d+A and A+A will help distangle different effects in HI systems

Summary and Perspectives

- Mean v_2 measurements
  - Large elliptic flow in Cu+Cu compared to Au+Au
- Eccentricity fluctuations
  - Participant eccentricity connects elliptic flow in small system to initial geometry fluctuations
  - Robustness studies wrt to changes in Glauber geometry
  - Work in progress with U.Heinz to investigate/check further properties
- Elliptic flow fluctuations
  - A method to measure elliptic flow fluctuations has been developed
  - Fluctuations in MC simulations are successfully reconstructed
  - Study systematic uncertainties due to MC/DATA differences
- Two-particle angular correlations
  - Properties of clusters in p+p, d+A, and A+A
  - Estimate non-flow contribution to elliptic flow fluctuations
    - Tune MC

Backup

Anisotropic flow

$$v_n = \langle \cos(n\phi - n\Psi_R) \rangle$$

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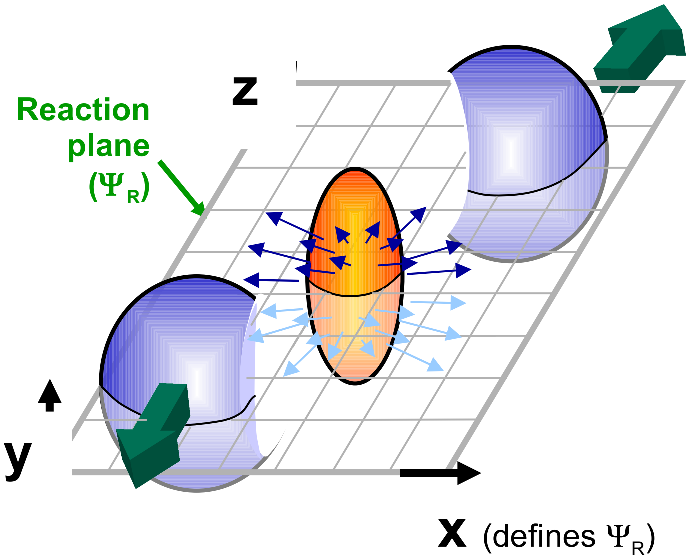
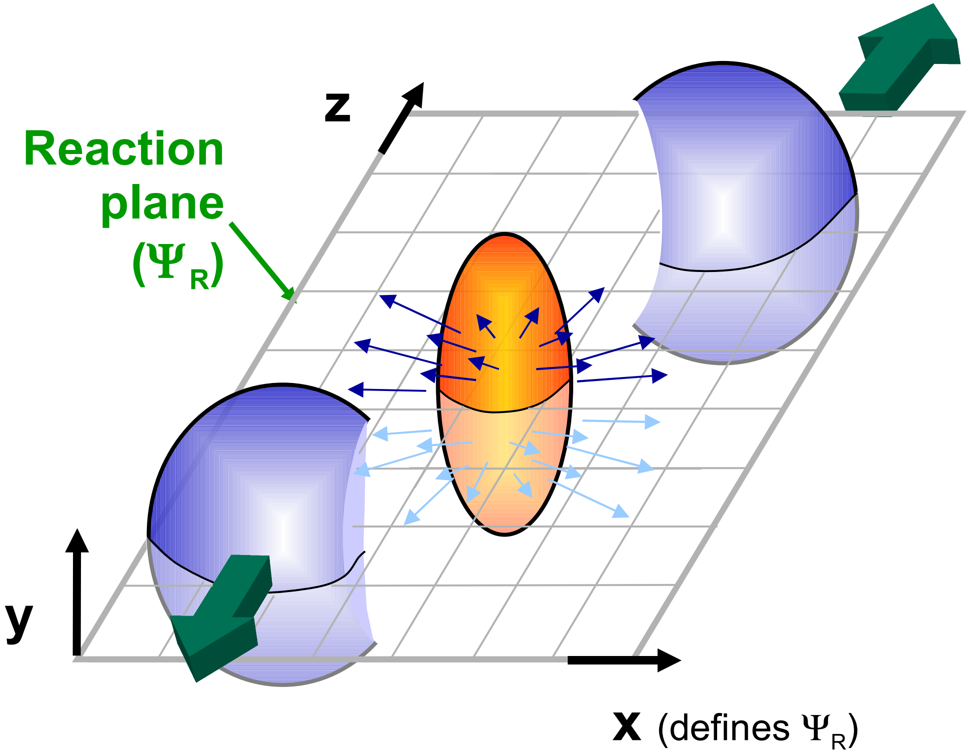
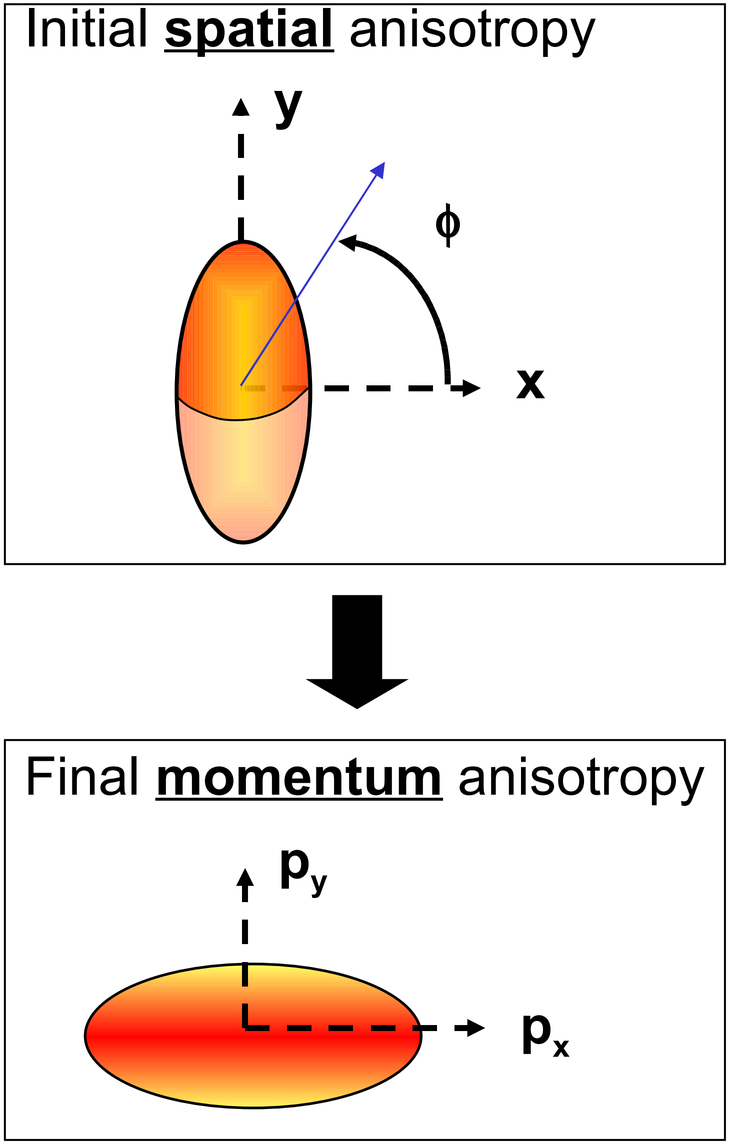


- Anisotropic flow ≡ azimuthal correlations with reaction plane
  - Consequence of thermalization
  - Final-state reinteractions
  - Typically described by hydro-dynamics (but flow does not necessary imply it)
- Non-flow ≡ contribution to v_n from azimuthal correlations between particles (HBT, resonances, jets)

Direct (v_1) and elliptic (v_2) flow



$$dN/d(\phi - \Psi_R) = N_0 (1 + 2v_1 \cos(\phi - \Psi_R) + 2v_2 \cos(2(\phi - \Psi_R)) + \ldots)$$


Initial **spatial** anisotropy

Final **momentum** anisotropy

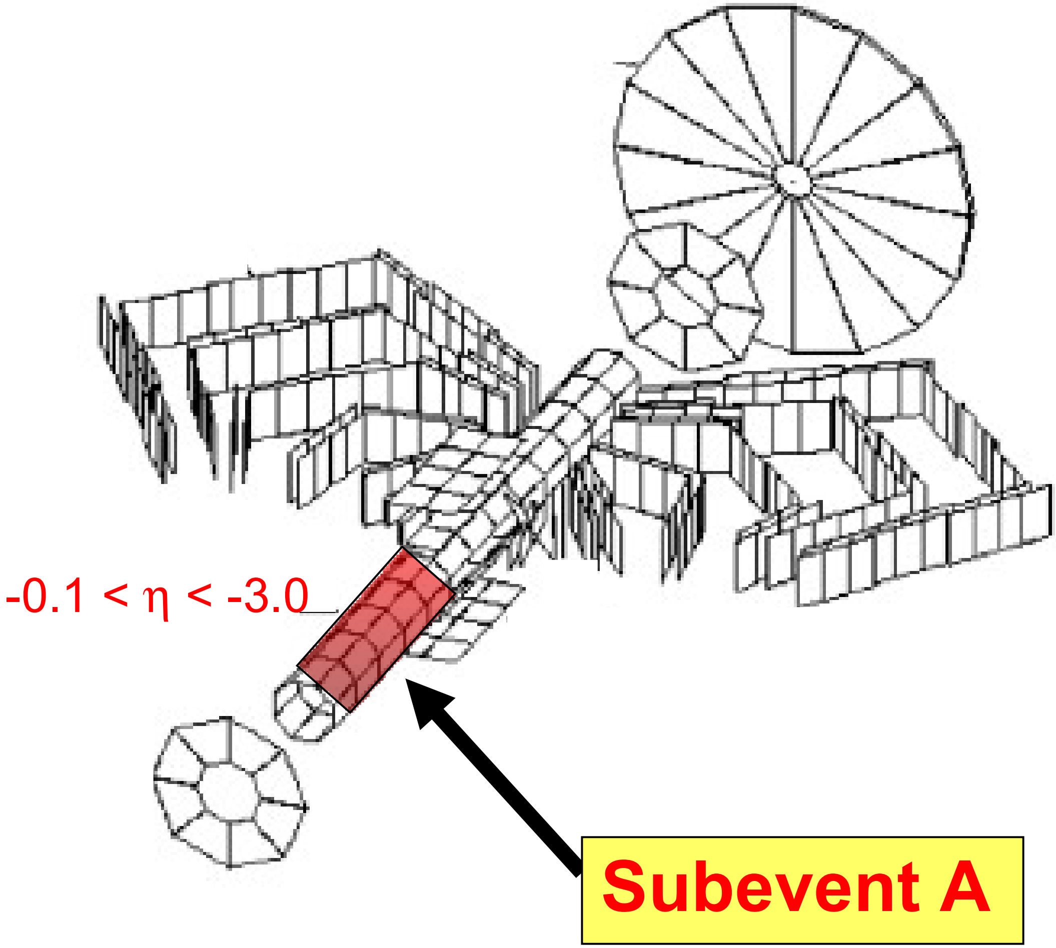
Flow measurement in PHOBOS

- Reaction-Plane / Subevent technique
  - Correlate reaction plane determined from azimuthal pattern of hits in one part of the detector with information from other parts of the detector
    - Hits
    - Tracks

$$\tan(2\psi_A) = \frac{\langle \sin(2\phi) \rangle_A}{\langle \cos(2\phi) \rangle_A}$$

$$v_2^{obs} = \langle \cos(2\phi - 2\psi_A) \rangle_B$$

$$v_2 = \frac{\langle v_2^{obs} \rangle_{events}}{\sqrt{\langle \cos(\psi_A - \psi_B) \rangle_{events}}}$$



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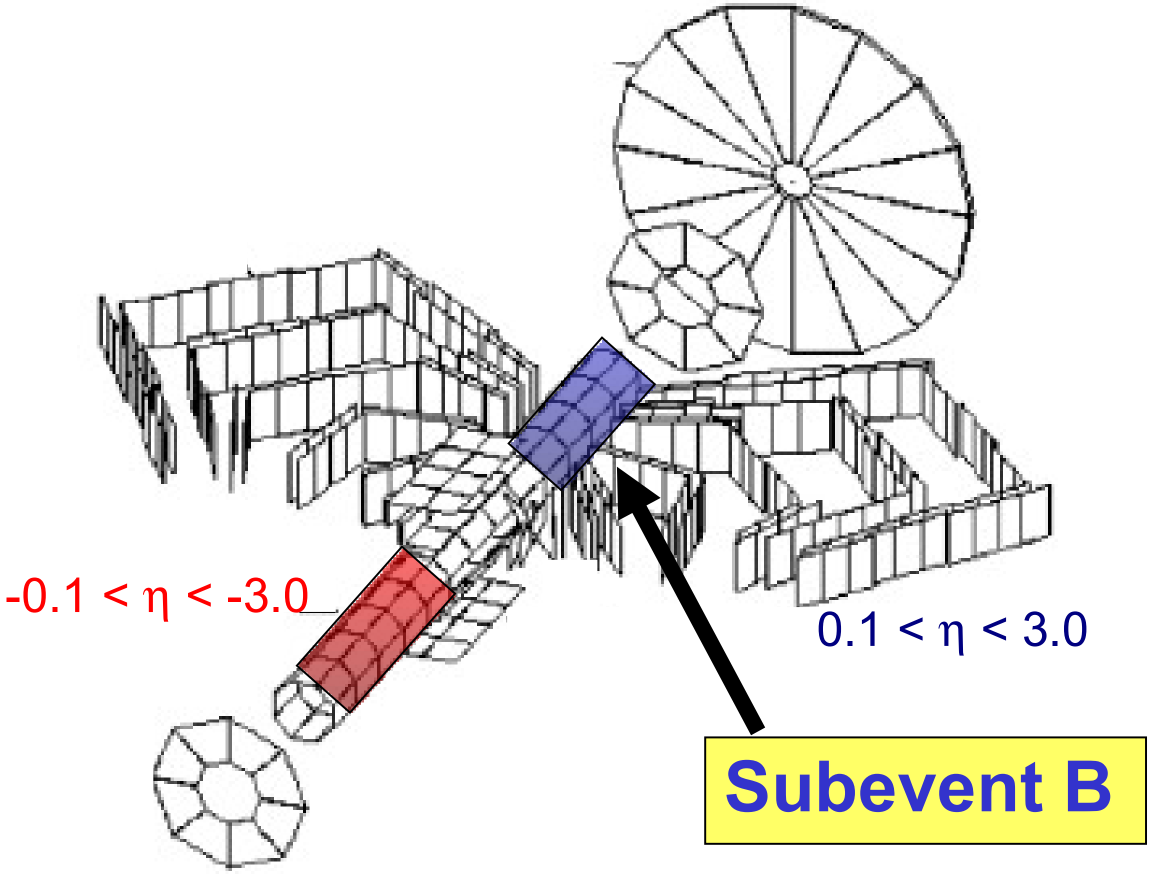

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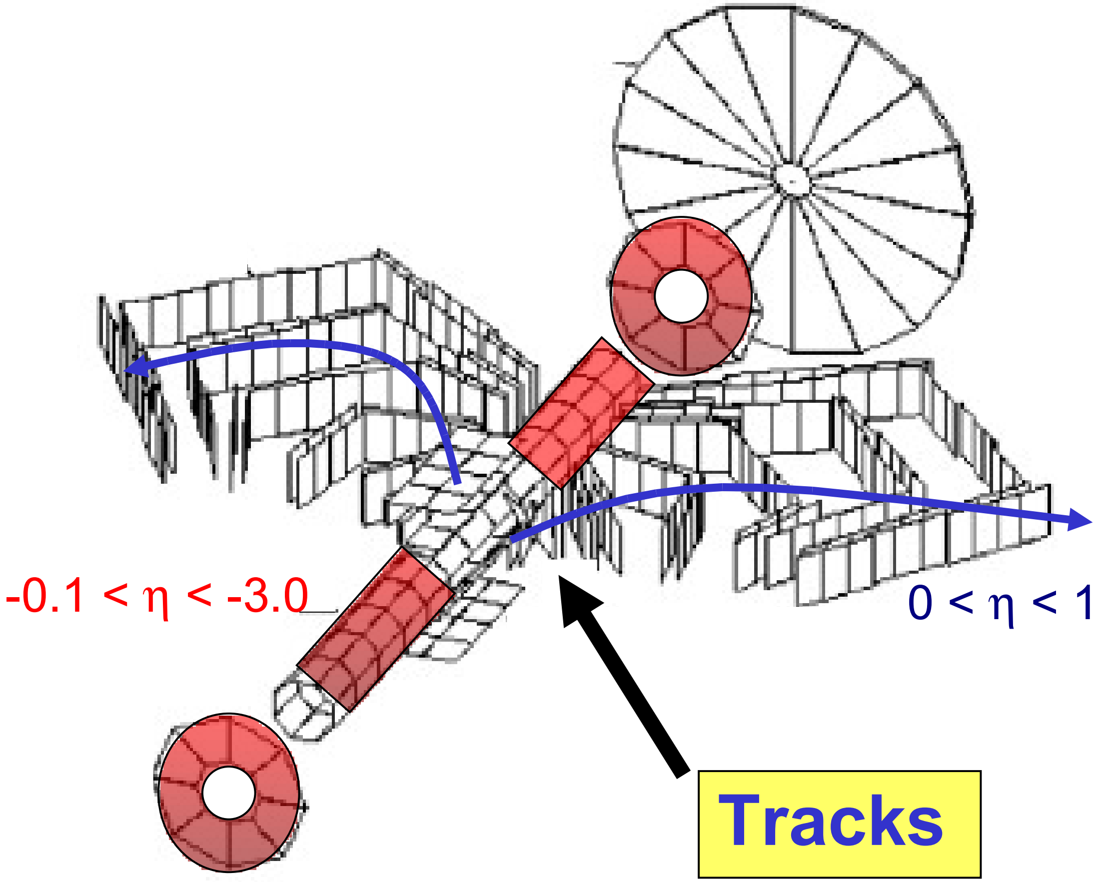

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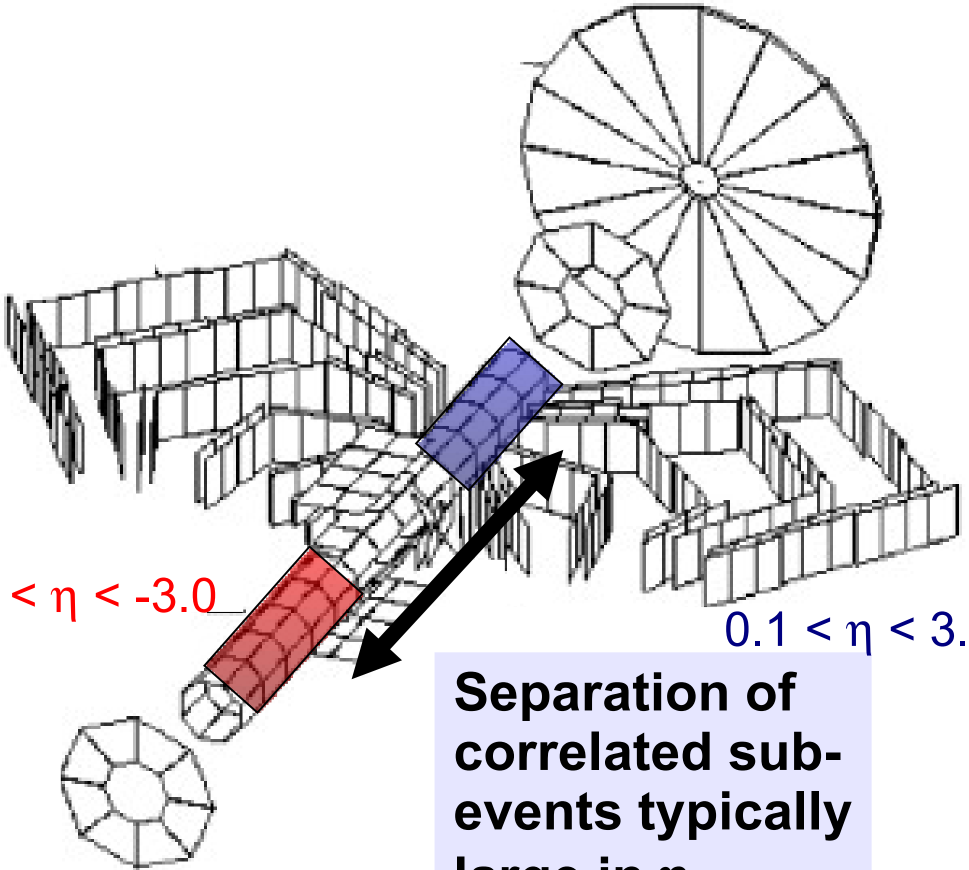
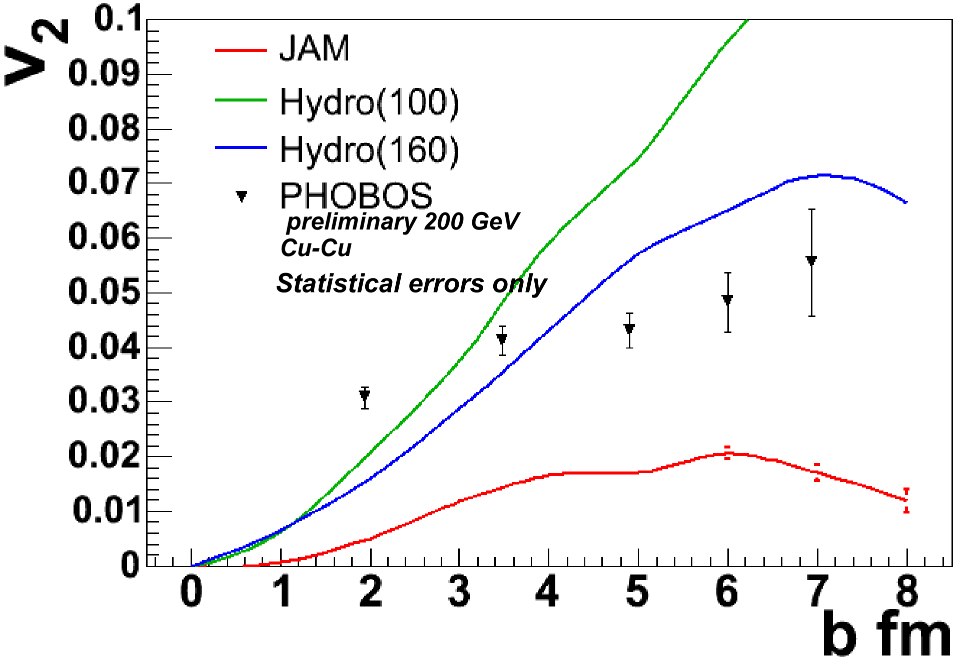
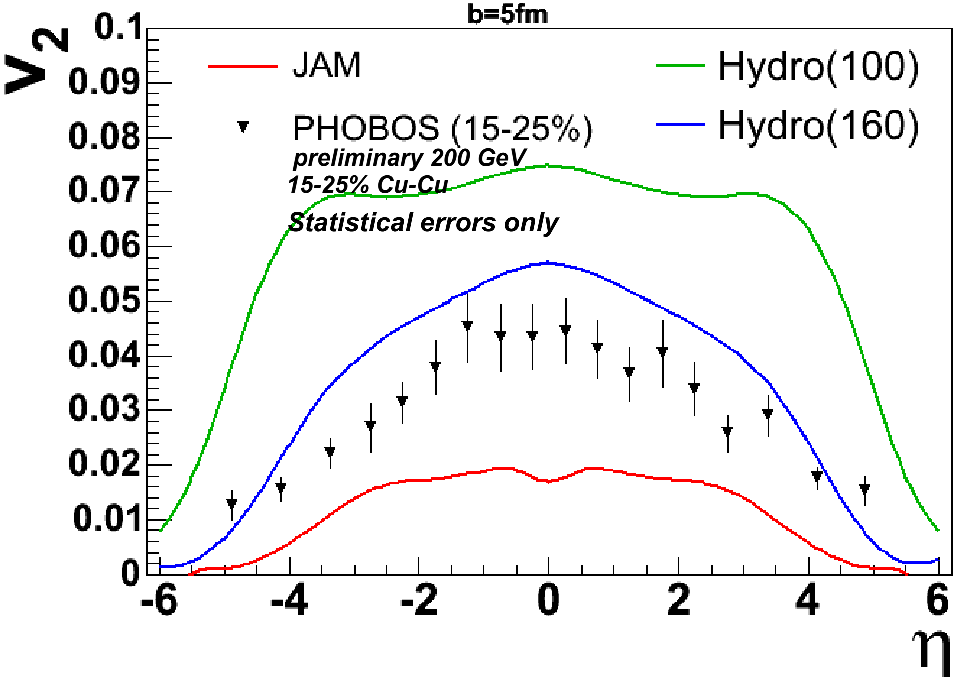


Separation of correlated sub-events typically large in η

Elliptic flow – Cu+Cu results



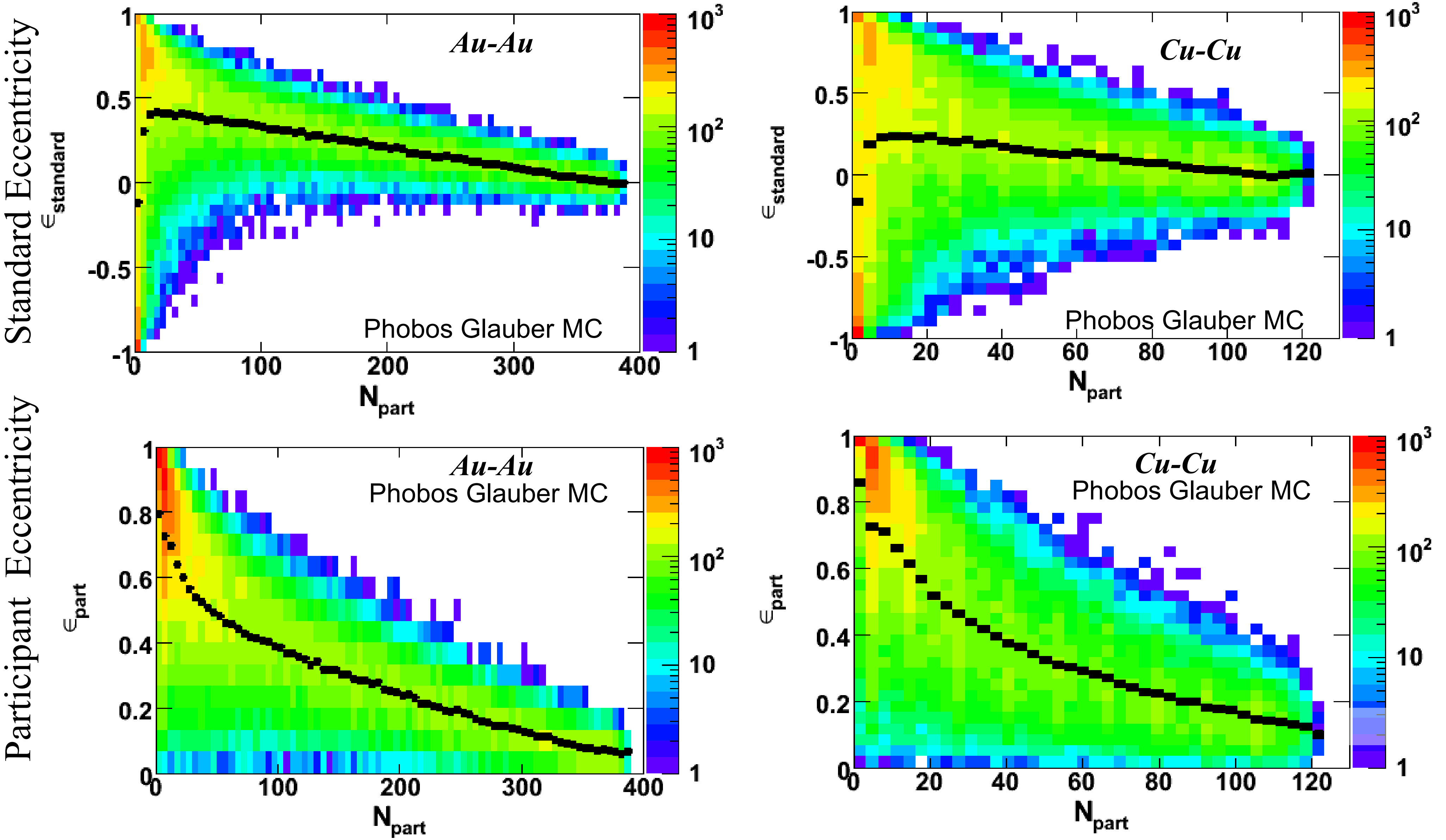


Cu+Cu more like Hydro than JAM hadron string cascade model

Models from Hirano et al., nucl-th/0506058; Here JAM uses a 1 fm/c formation time. Hydro (160) has kinetic freezeout temperature at 160 MeV

Standard and participant eccentricity

Mean eccentricity shown in black

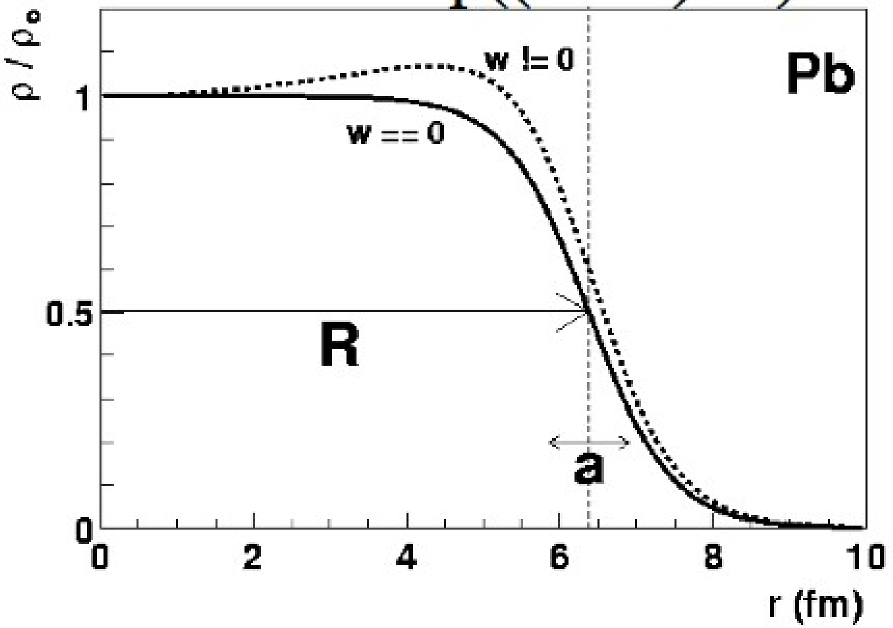


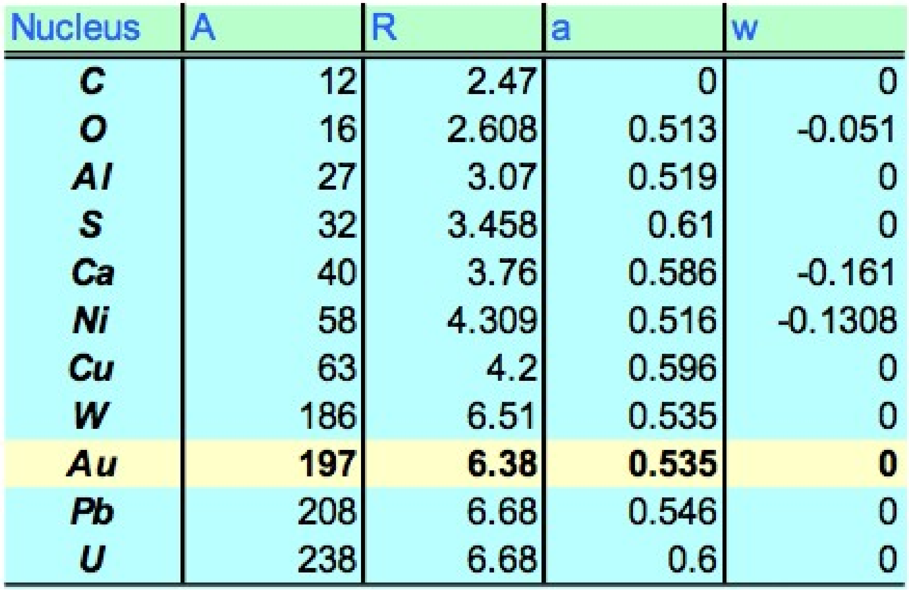
Greater fluctuations in Cu+Cu. Positive fluctuations lead to non zero mean.

Glauber Parameters Changed

Systematic Source	Standard	How Much We Vary
Nucleon-nucleon cross-section	42 mb (for 200GeV)	30 mb (<20GeV) 45 mb (>200GeV)
Nuclear skin depth	0.535fm(Au)0.596fm(Cu)	±10%
Nuclear radius	6.38fm (Au)4.2fm (Cu)	±10%
Minimum nucleon separation (center-to-center)	0.4fm (like HIJING)	0fm 0.8fm

$$\rho(r) = \frac{\rho_0\left(1 + wr^2 / R^2\right)}{1 + \exp((r - R)/a)}$$

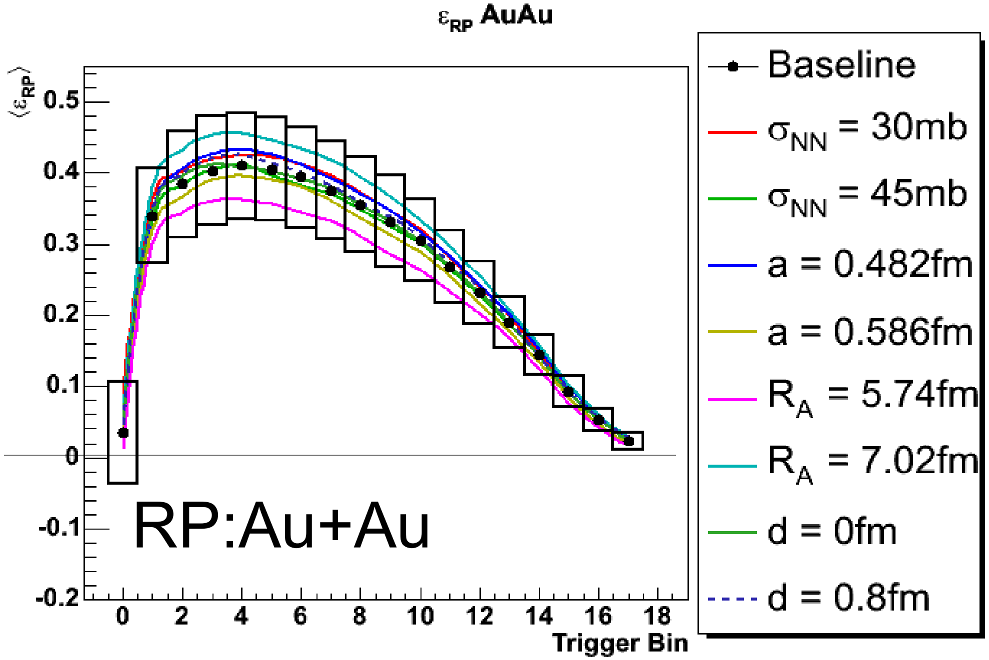


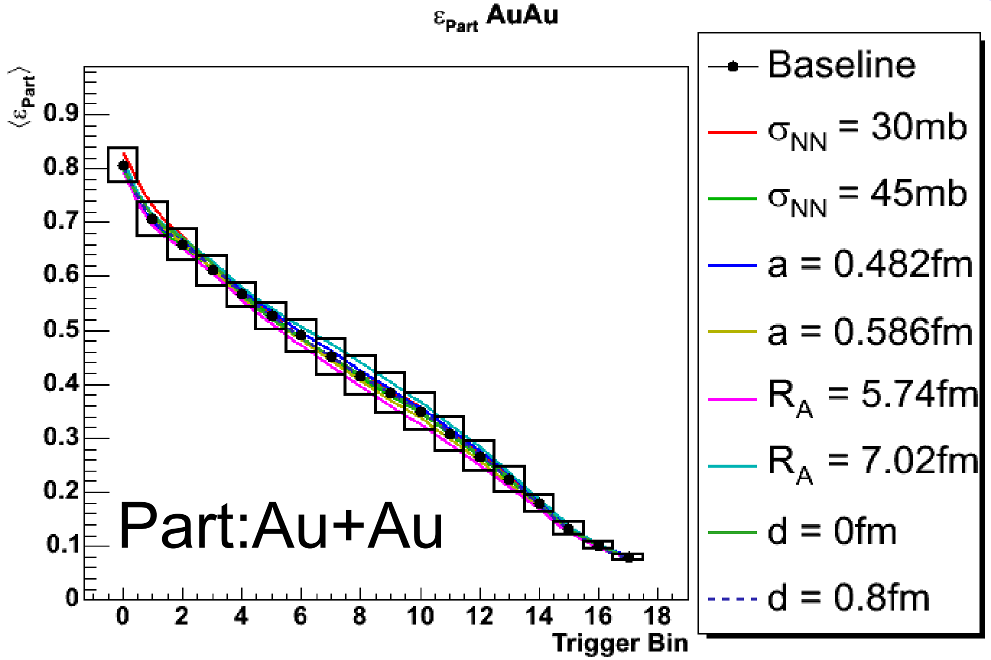


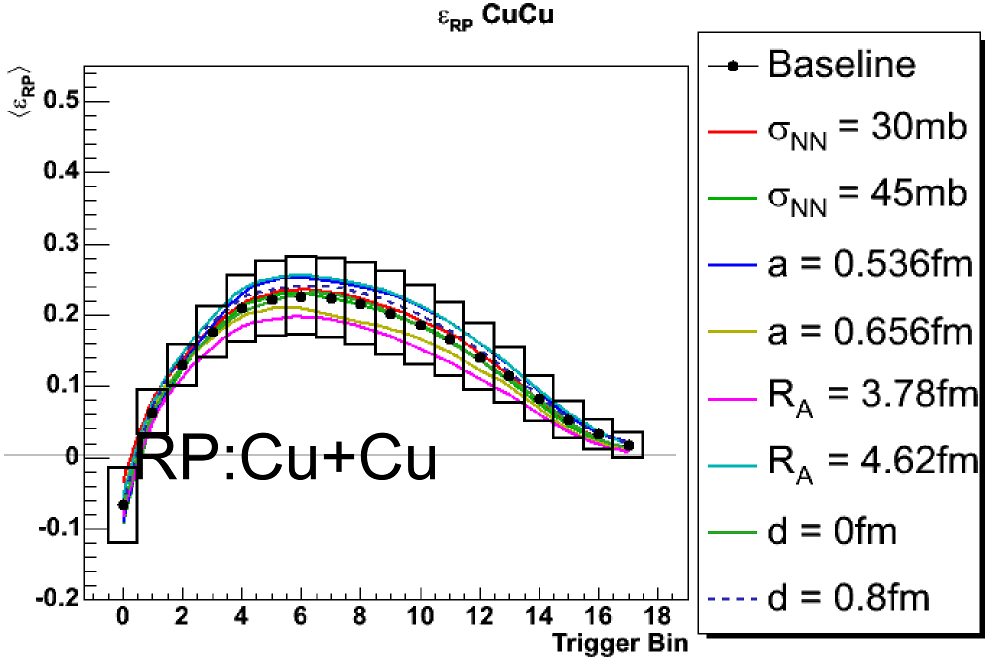
Nucleus	A	R	a	w
C	12	2.47	0	0
O	16	2.608	0.513	-0.051
Al	27	3.07	0.519	0
S	32	3.458	0.61	0
Ca	40	3.76	0.586	-0.161
Ni	58	4.309	0.516	-0.1308
Cu	63	4.2	0.596	0
W	186	6.51	0.535	0
Au	**197**	**6.38**	**0.535**	**0**
Pb	208	6.68	0.546	0
U	238	6.68	0.6	0

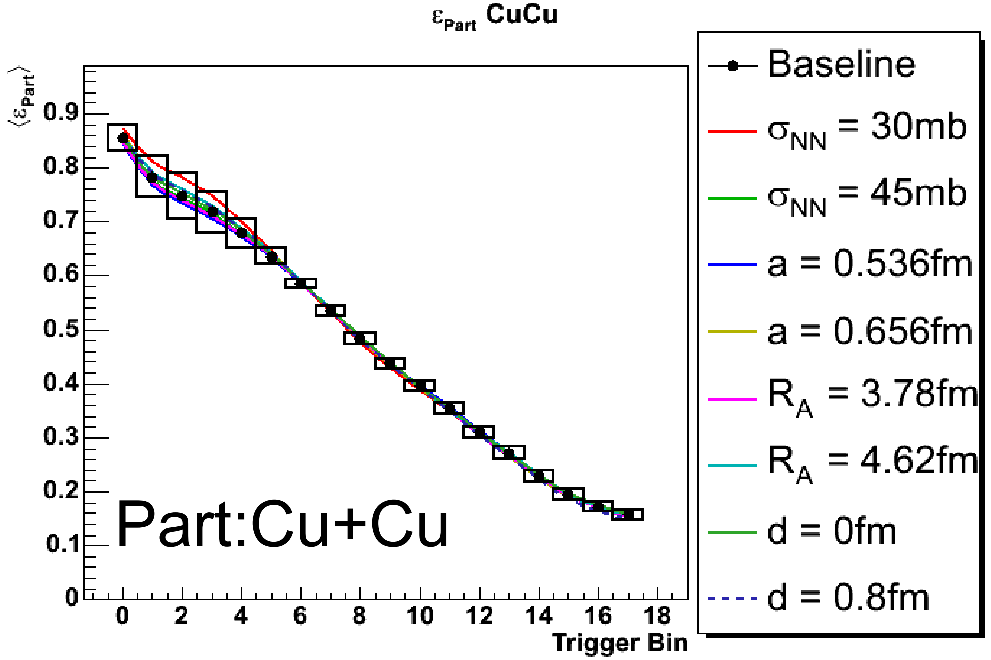
H. DeVries, C.W. De Jager, C. DeVries, 1987

Robustness with geometry variables

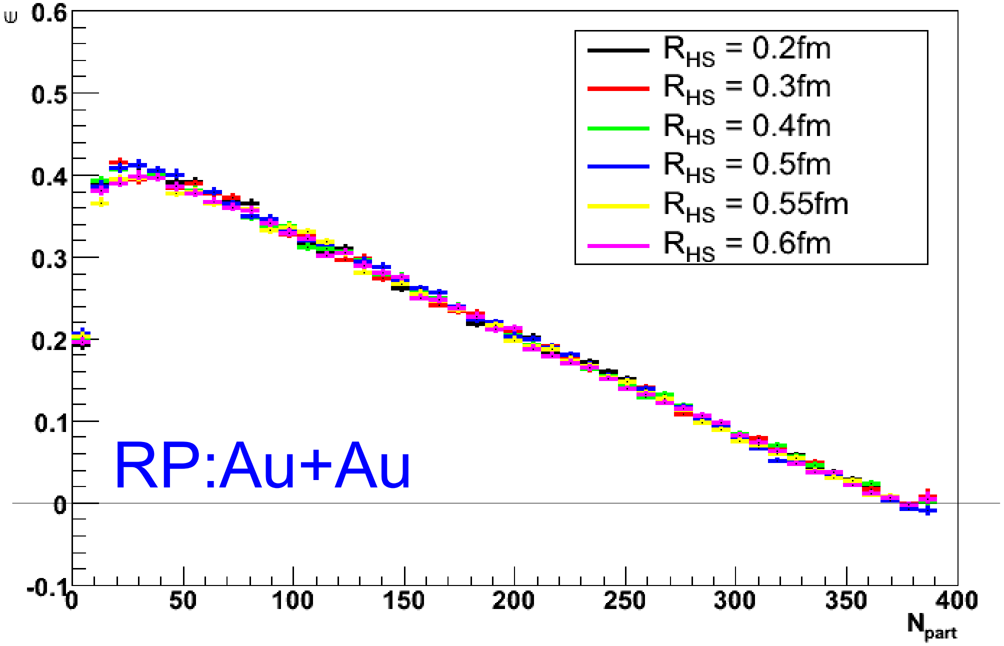


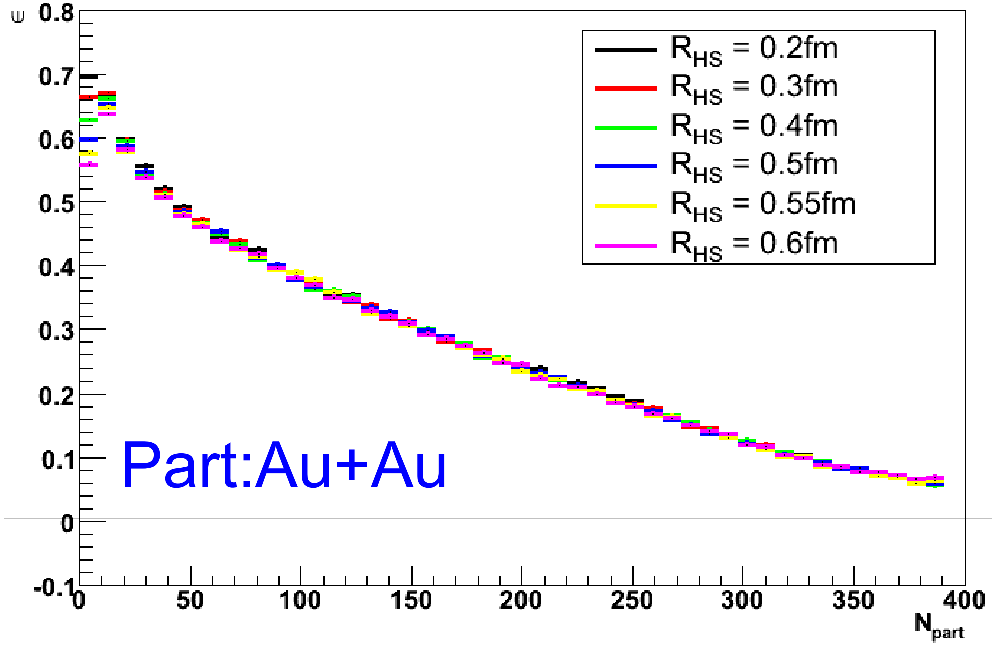


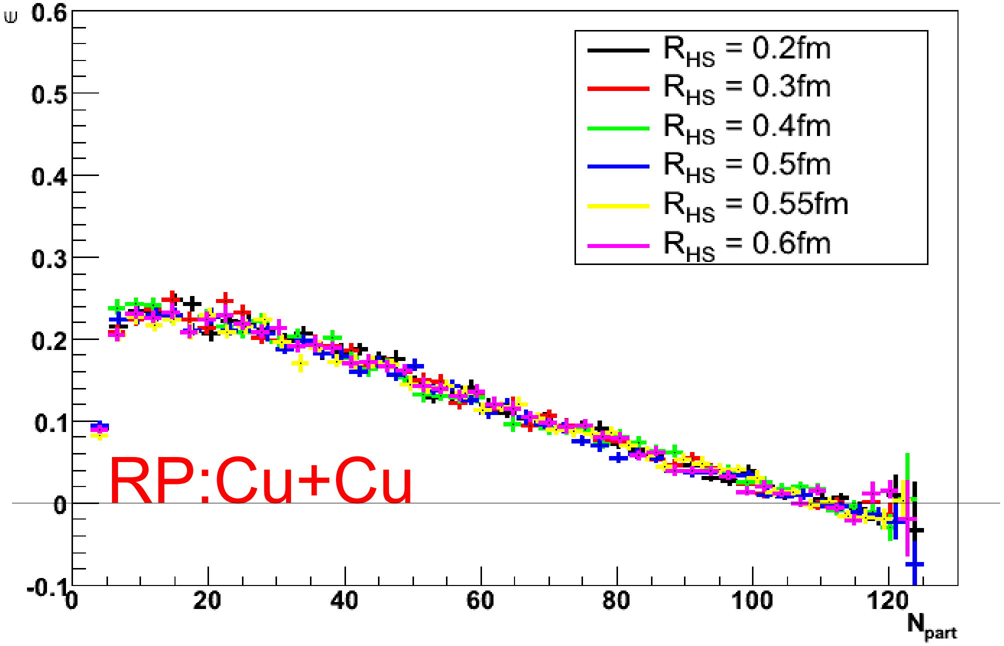


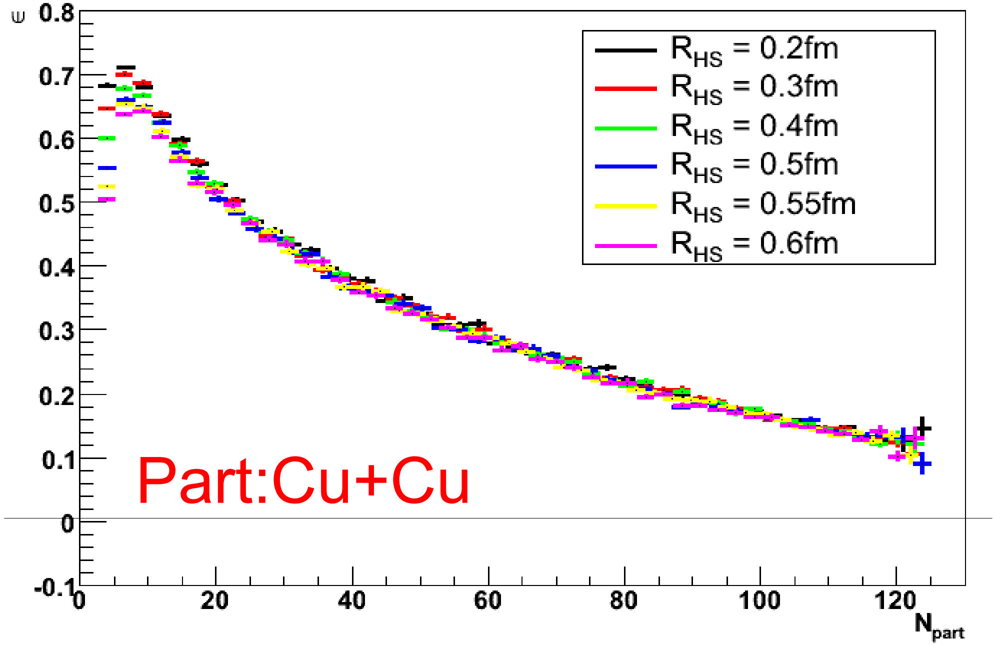


Hard-sphere smearing (Npart weighted)

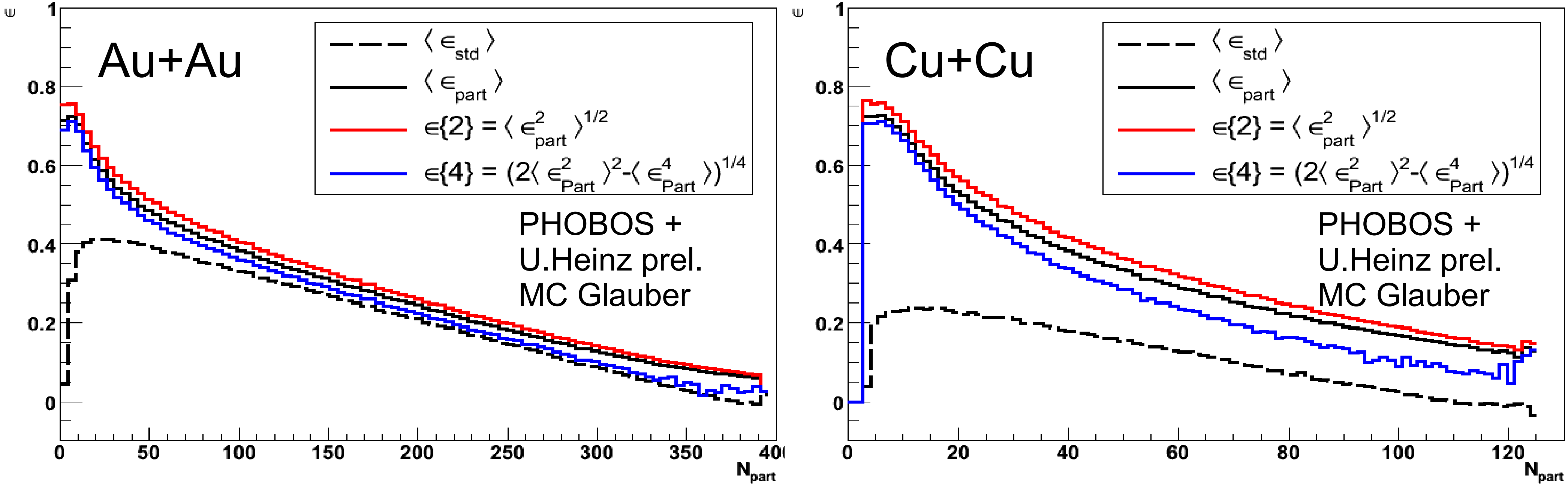








Moments of eccentricity

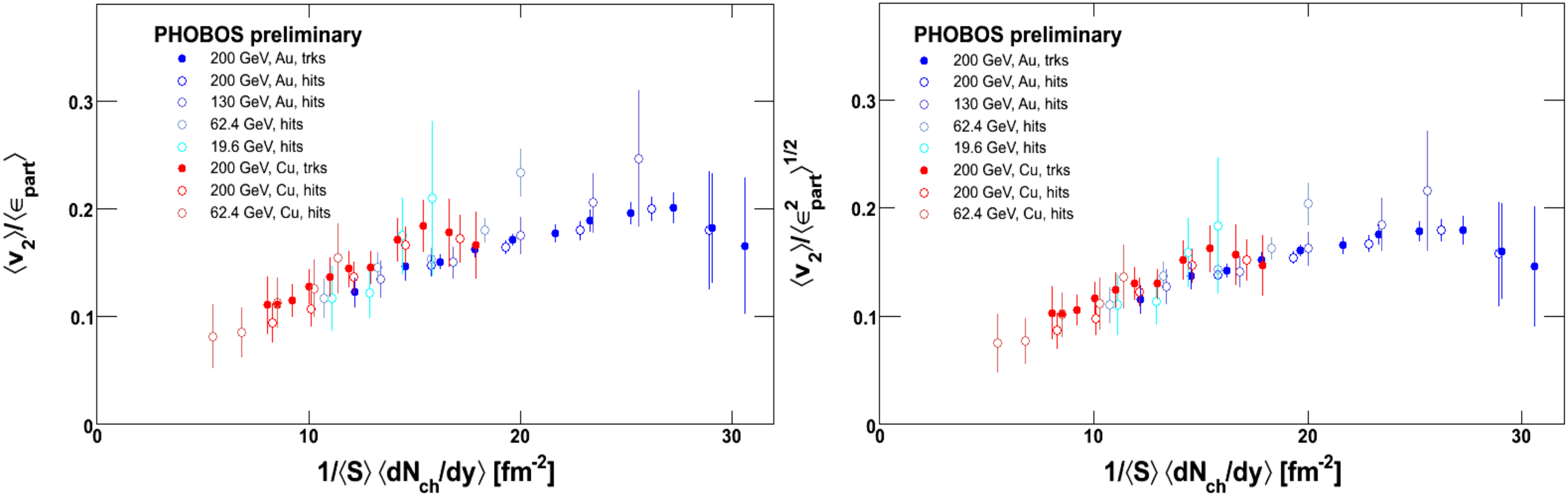


- If v_2 fluctuates proportional to ϵ_{part}, then use cumulants for v_2/ϵ ratio
  - $\epsilon\{2\}$ rather similar to ϵ_{part}, unlike $\epsilon\{4\}$ that is between ϵ_{std} and ϵ_{part}
- Outstanding question:
  To what extent is PHOBOS $v_2\{RP\}$ sensitive to higher moments in v2
  - Answer probably dependent on system

M.Miller, R.Snellings, nucl-ex/0312008
R.Bhalerao, J.Ollitrault, nucl-th/0607009

Voloshin plots wrt to participant eccentricity

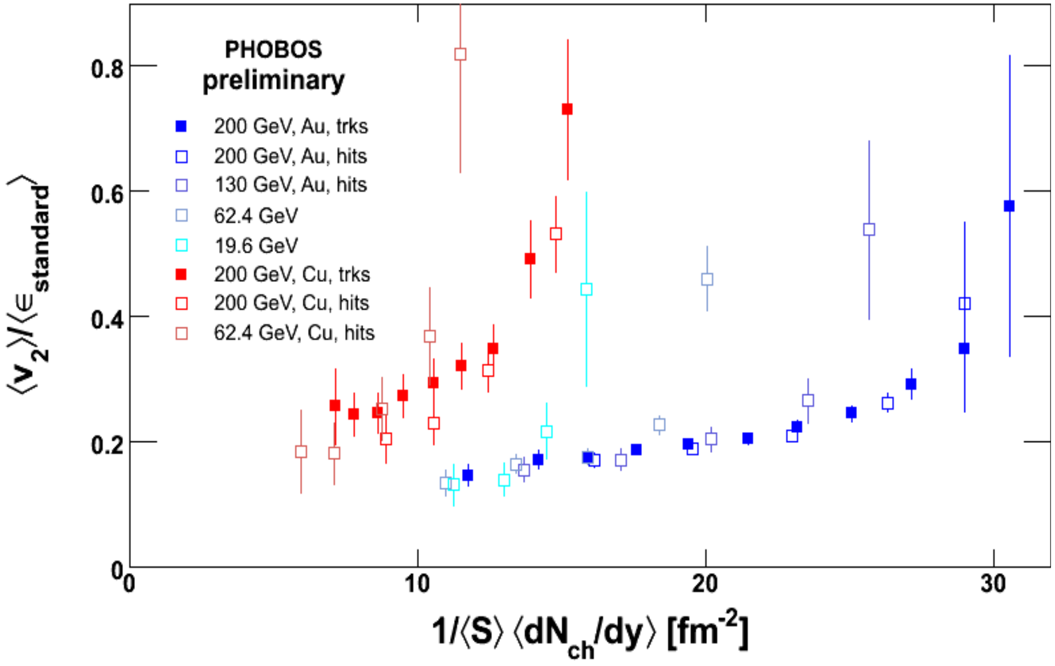
- Open question (issue): v2{RP} ≈ v2{2} (claimed by Star, Bhalerao/Ollitrault)

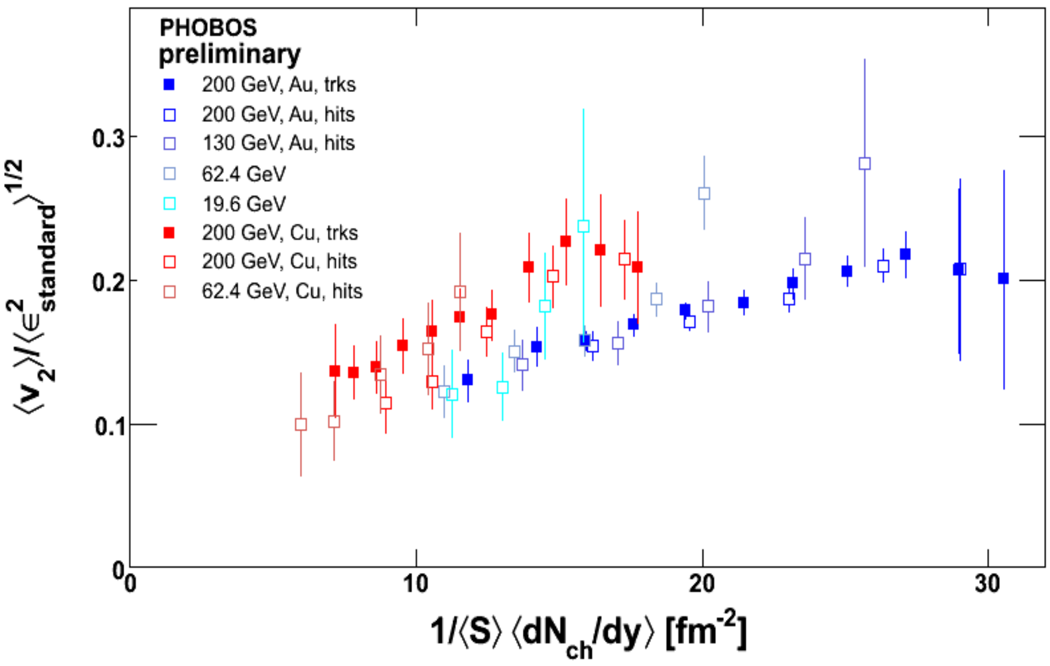


- Quantitative "improvement" of the scaling

Voloshin plots wrt to reaction-plane eccentricity

- Open question (issue): v2{RP} ≈ v2{2} (claimed by Star, Bhalerao/Ollitrault)





- Qualitative “improvement” of the scaling ?

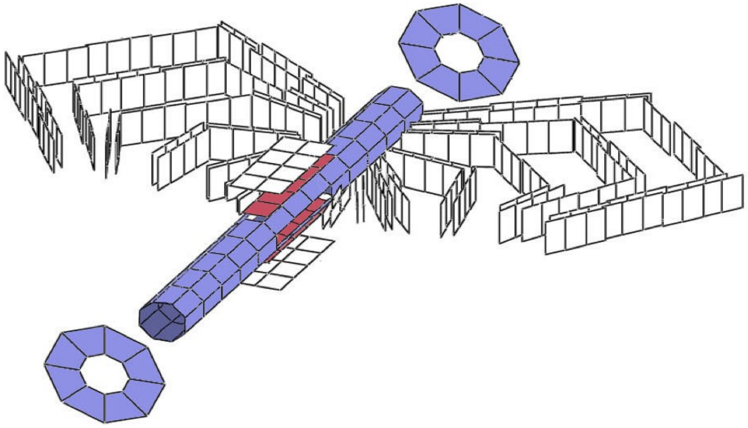
Event-by-event measurement of v_2^{obs}

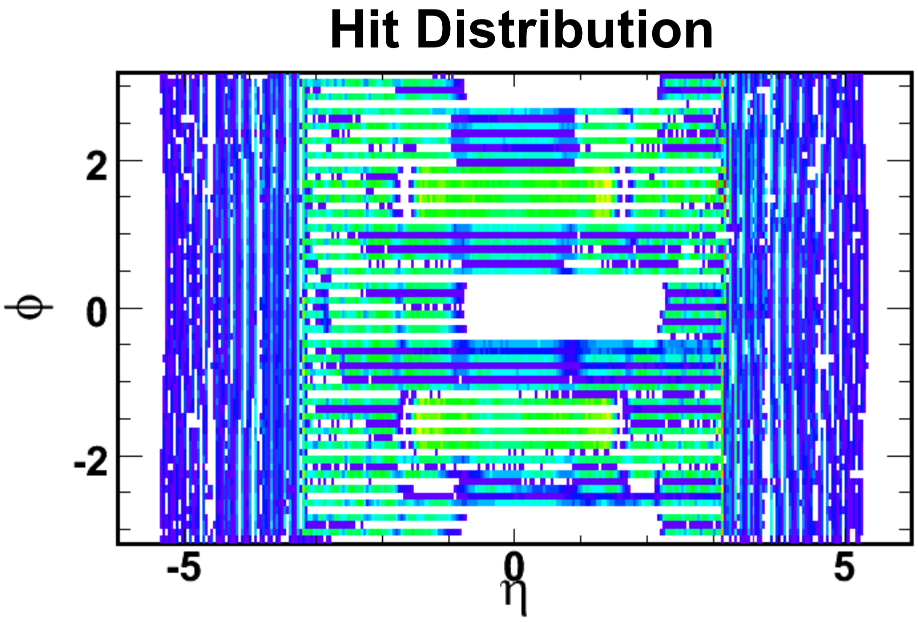
- PHOBOS Multiplicity Array

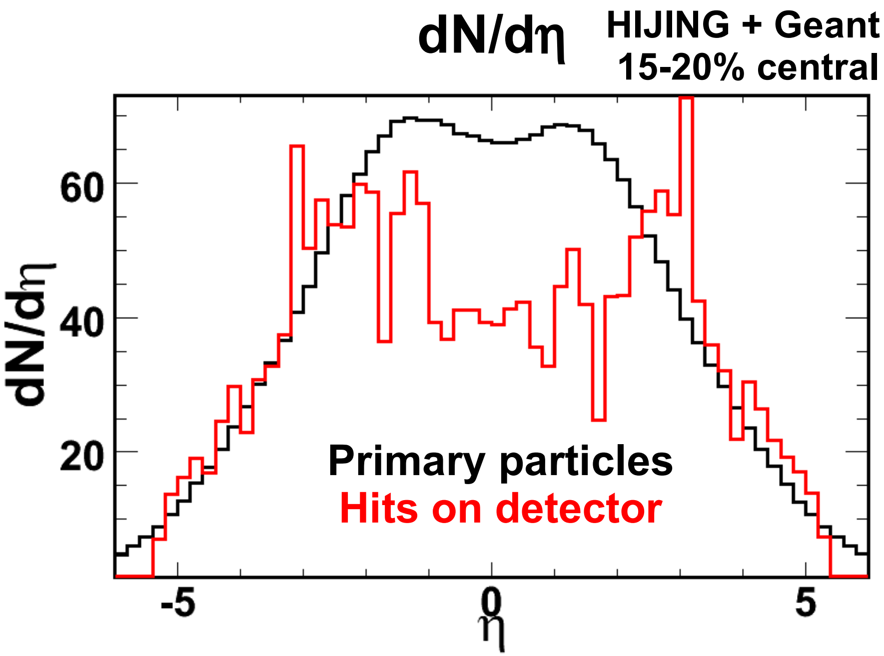
  -5.4<η<5.4 coverage

  -Holes / granularity differences

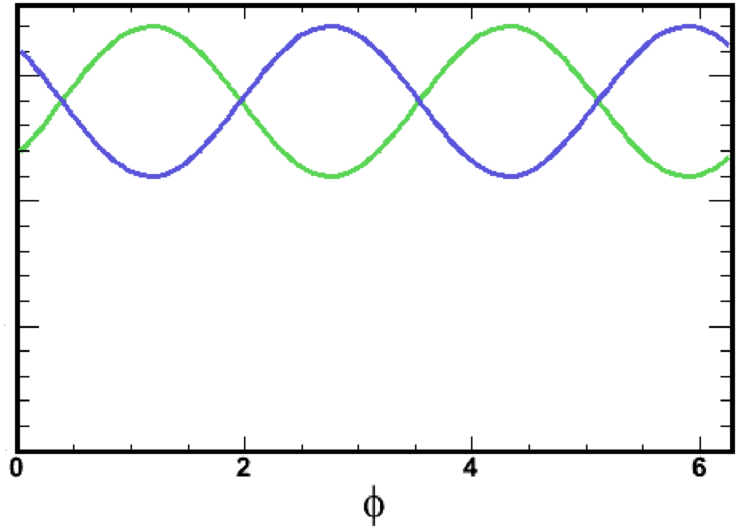
- Usage of all available information in event to determine single value for v_2^{obs}

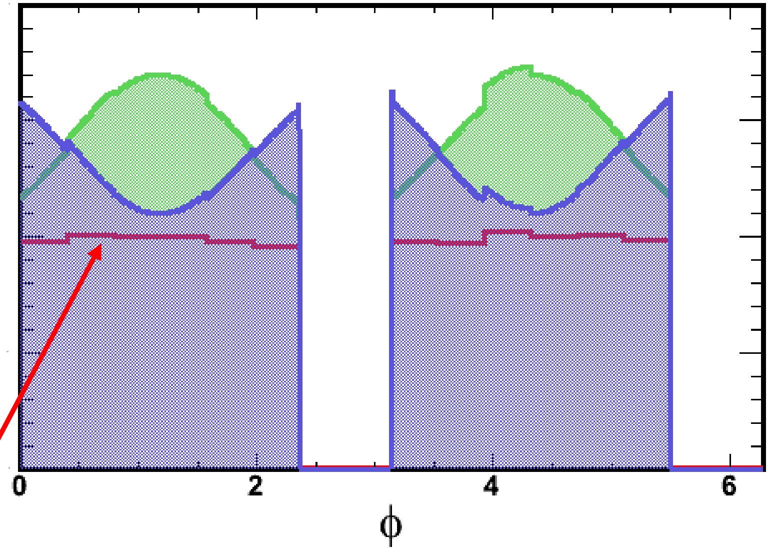






Likelihood fit normalization





Acceptance

$$s(u,\phi_0|\eta) = \int \boxed{A(\eta,\phi)} \cdot [1 + 2\,u\,(1 - |\eta|/6)\,cos(2(\phi - \phi_0))]\,d\phi\,d\eta$$

$$L(u,\phi_0) = \prod_{i=1}^{n} \frac{1}{s(u,\phi_0|\eta_i)}\,[1 + 2\,u\,(1 - |\eta_i|/6)\,cos(2(\phi_i - \phi_0))]$$

Event-by-event measurement of v_2^{obs}

$$L(v_2^{obs},\phi_0)=\prod_{i=1}^{n} p(\eta_i)\left[1+2v_2^{obs}\left(1-\frac{|\eta_i|}{6}\right)\cos(2\phi-2\phi_0)\right]$$

- Maximize likelihood to find "most likely" value of v_2^{obs} *(and* ϕ_0*)*
- Comparing values of v_2^{obs} and ϕ_0
  - PDF folded by acceptance must be normalized to the same value for different v_2^{obs} and ϕ_0's

$$p(v_2^{obs},\phi_0;\eta)=1/\int_{\eta} A(\eta,\phi)\left[1+2v_2^{obs}\left(1-\frac{|\eta|}{6}\right)\cos(2\phi-2\phi_0)\,d\eta\, d\phi\right]$$

Acceptance

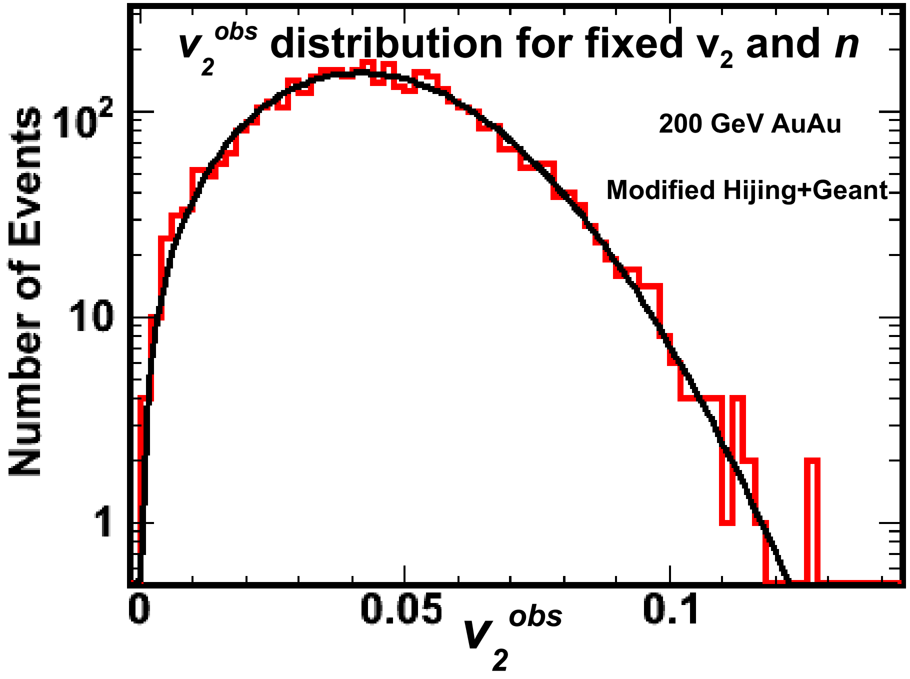
Determining the kernel

- In a single bin of v_2 and *n*

$$K(v_2^{obs};\ v_2, n) = v_2^{obs} \exp\left(\frac{-(v_2^{obs} - a)^2}{2b^2}\right)$$

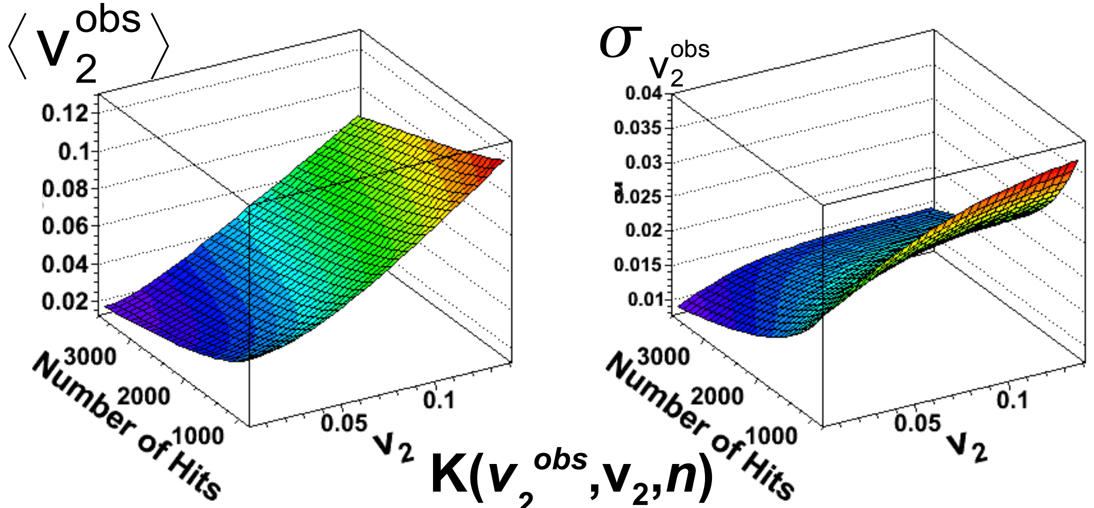
  - Correspondance

$$(a,b) \Leftrightarrow (\langle v_2^{obs} \rangle, \sigma_{v_2^{obs}})$$



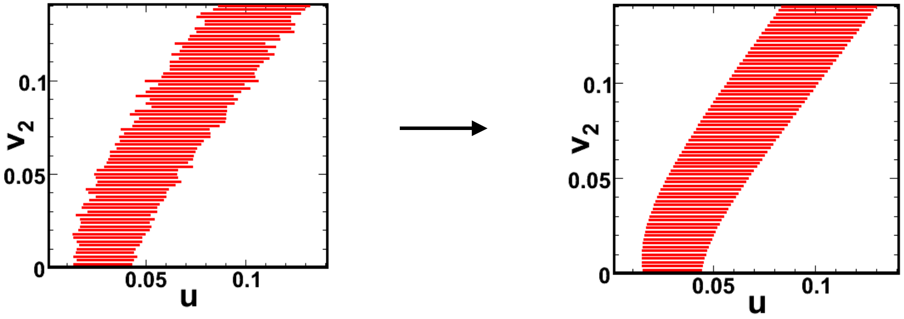
- Determine $\langle v_2^{obs} \rangle$ and $\sigma_{v_2^{obs}}$ in bins of v_2 and n and determine (a,b)
- Fit smooth functions
- Integrate out multiplicity

$$K(v_2^{obs};\ v_2) = \int K(v_2^{obs};\ v_2, n)\, N(n)\, dn$$



$K(v_2^{obs}, v_2, n)$

Calculating the kernel: Functions observed to fit

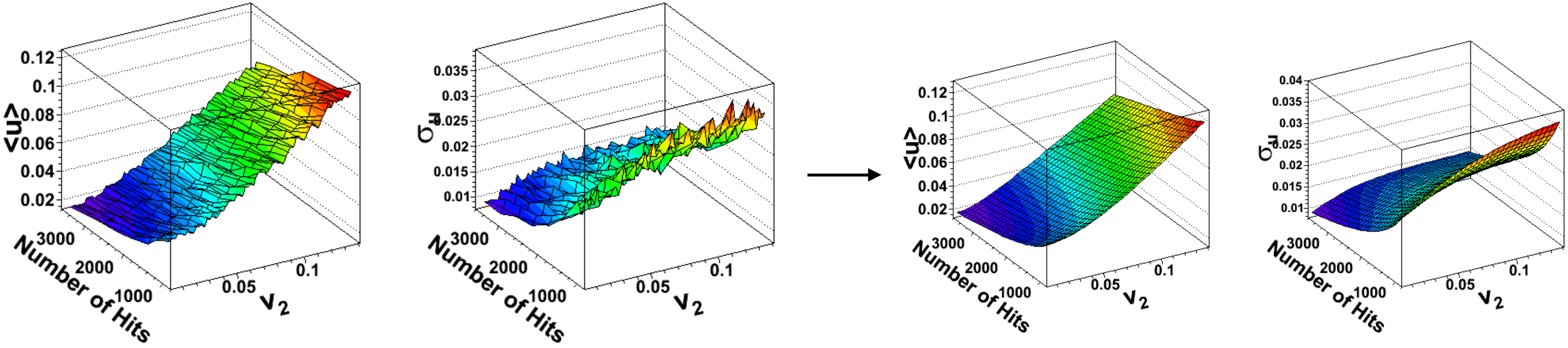


$$< u > (v_2)|_n = \sqrt{m_1 \cdot v_2^2 + m_2}$$

$$\sigma_u(v_2)|_n = \frac{1}{r_1 + \frac{2}{3}\, e^{r_2 v_2}}$$

$$< u > (v_2, n) = \sqrt{(M_1 n^2 + M_2 n + M_3) \cdot v_2^2 + (M_4 * n + M_5)}$$

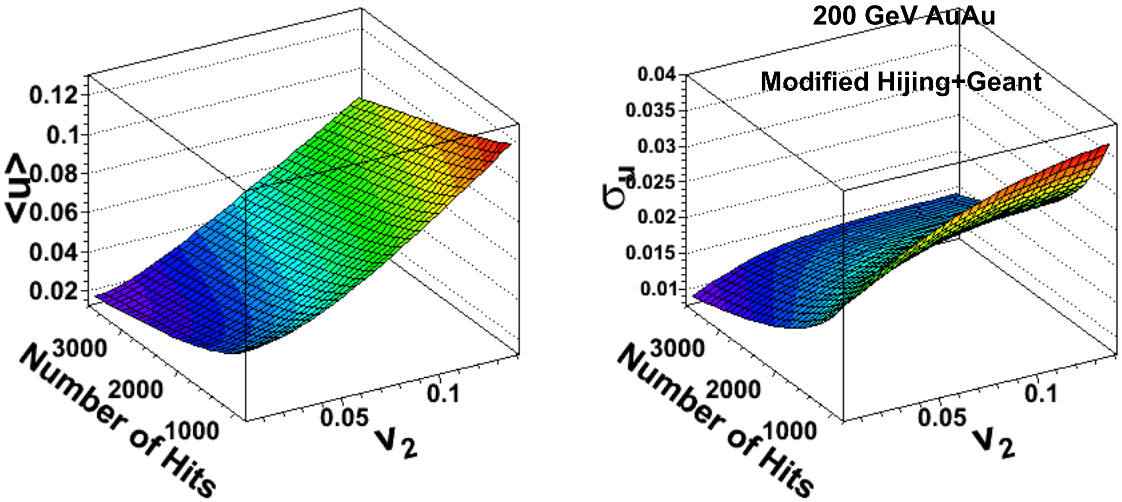
$$\sigma_u(v_2, n) = \frac{R_1}{(R_2\sqrt{n} + 1) \cdot (1 + \frac{2}{3}\, e^{(R_3 n + R_4) v_2})}$$



Determining the kernel

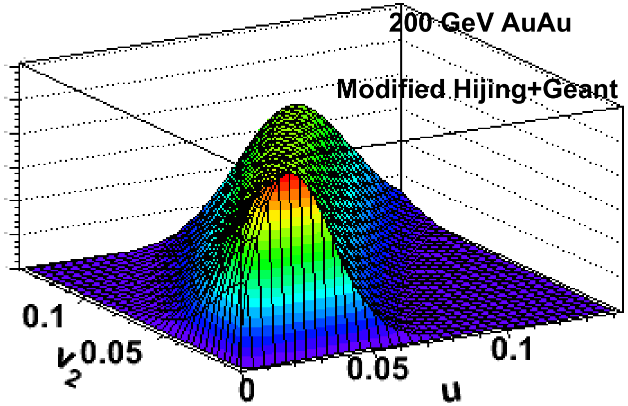
- Multiplicity dependence can be integrated out

K(*u*,v$_2$,*n*)

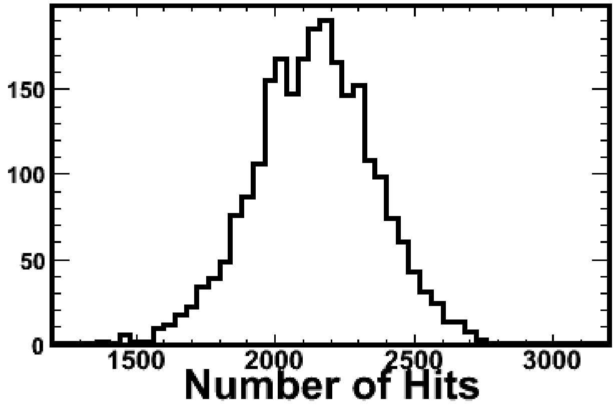


$$K(v_2^{obs};\, v_2) = \int K(v_2^{obs};\, v_{2,} n)\, N(n) dn$$

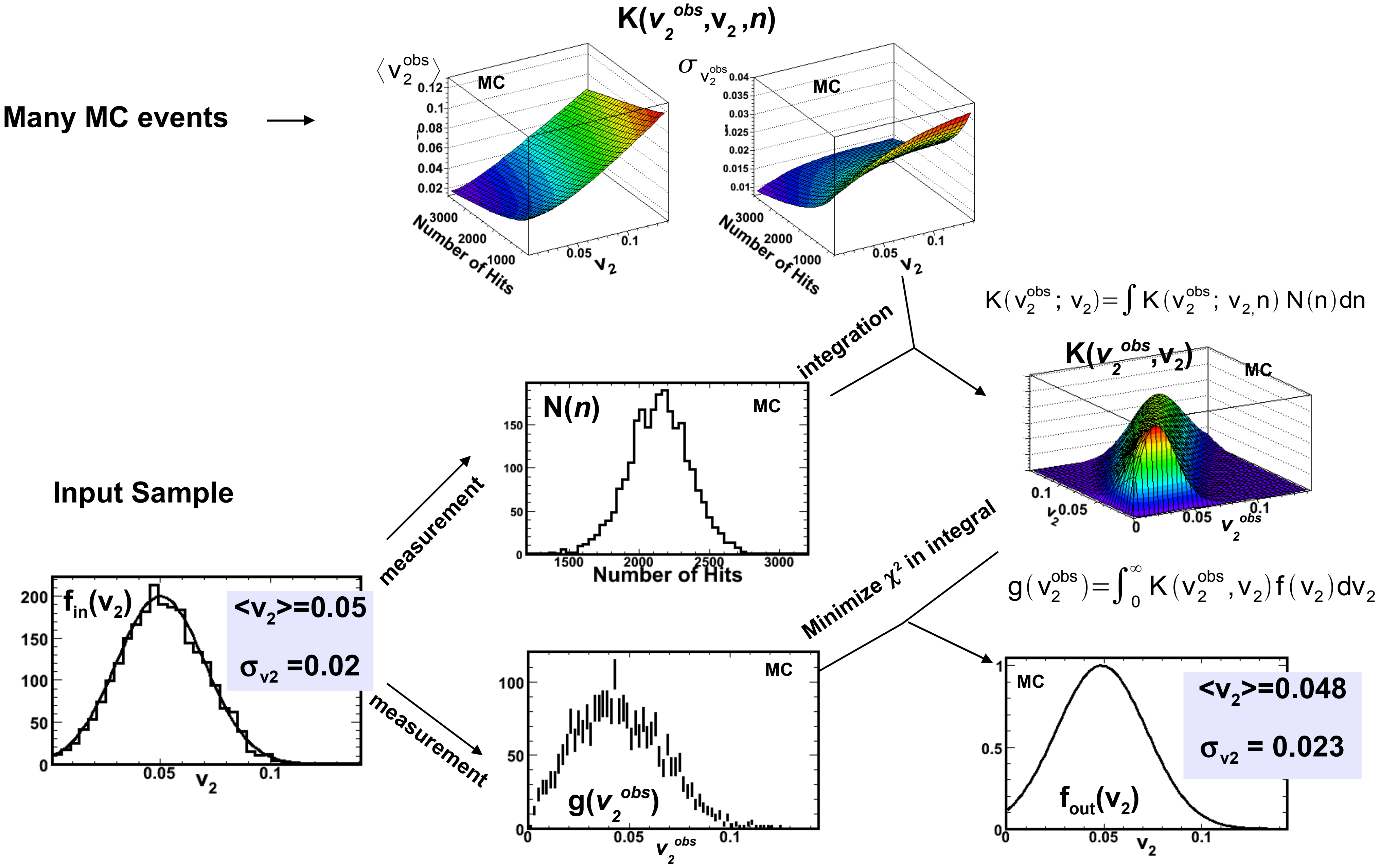
K(*v*$_2$^{obs},v$_2$)



N(*n*) = Number hits distribution in sample



Summary of flow fluctuation measurement method



Many MC events →

$K(v_2^{obs}, v_2, n)$

$K(v_2^{obs}; v_2) = \int K(v_2^{obs}; v_2, n)\, N(n)\, dn$

integration

Input Sample

$\langle v_2 \rangle = 0.05$

$\sigma_{v2} = 0.02$

measurement

measurement

Minimize χ^2 in integral

$g(v_2^{obs}) = \int_0^{\infty} K(v_2^{obs}, v_2)\, f(v_2)\, dv_2$

$\langle v_2 \rangle = 0.048$

$\sigma_{v2} = 0.023$