

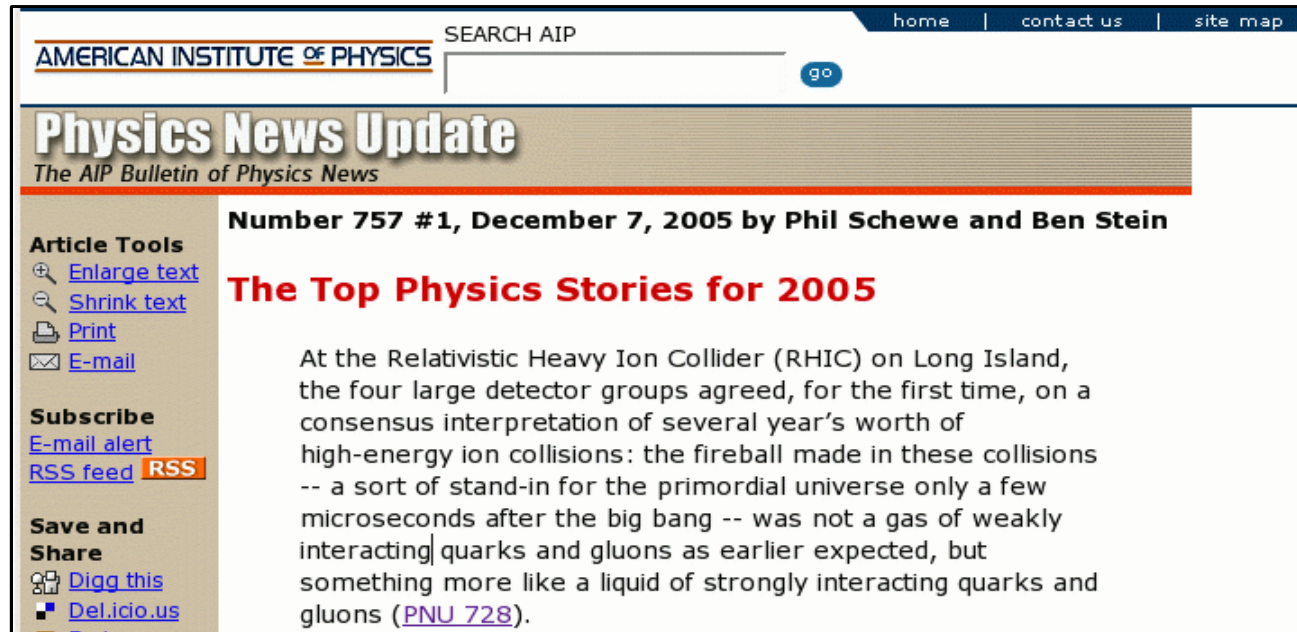
Importance of correlations and fluctuations on the initial eccentricity

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Minisymposium on Flow, Eccentricity and the Equation of State in RHIC

April 13, 2008, APS meeting, St Louis, MO

AIP Top Physics Story, Dec 2005



The screenshot shows the AIP website interface. At the top, there is a search bar with the text "SEARCH AIP" and a "go" button. Below the search bar, the AIP logo is visible. The main heading is "Physics News Update" with the subtitle "The AIP Bulletin of Physics News". The article title is "Number 757 #1, December 7, 2005 by Phil Schewe and Ben Stein" and "The Top Physics Stories for 2005". The article text begins with "At the Relativistic Heavy Ion Collider (RHIC) on Long Island, the four large detector groups agreed, for the first time, on a consensus interpretation of several year's worth of high-energy ion collisions: the fireball made in these collisions -- a sort of stand-in for the primordial universe only a few microseconds after the big bang -- was not a gas of weakly interacting quarks and gluons as earlier expected, but something more like a liquid of strongly interacting quarks and gluons (PNU 728)." The left sidebar contains "Article Tools" (Enlarge text, Shrink text, Print, E-mail), "Subscribe" (E-mail alert, RSS feed), and "Save and Share" (Digg this, Del.icio.us).

“... the fireball made in these [heavy-ion] collisions ... was not a gas of weakly interacting quarks and gluons as earlier expected, but **something more like a liquid** of strongly interacting quarks and gluons”

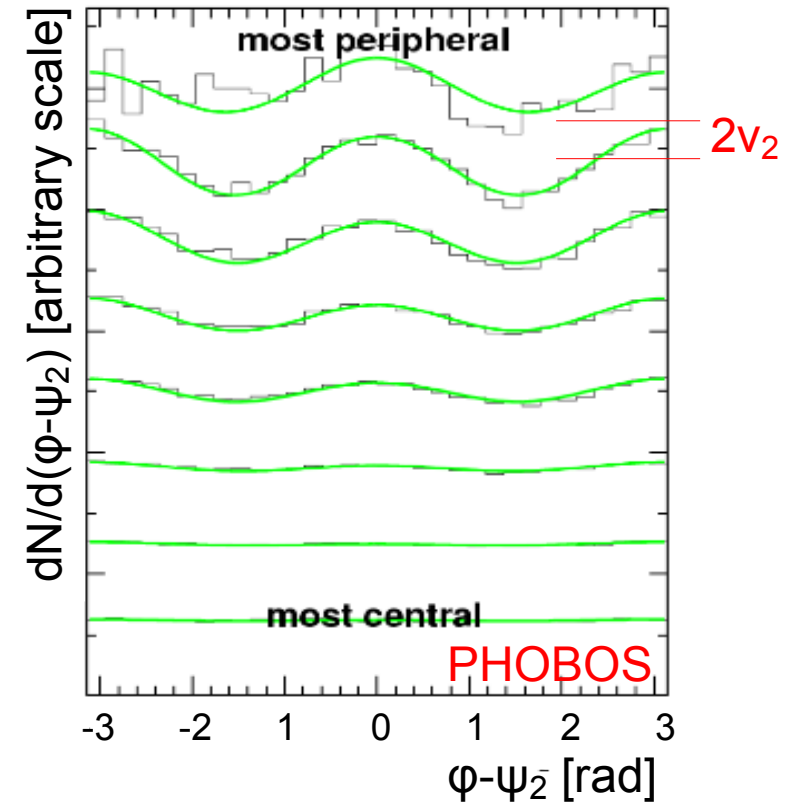
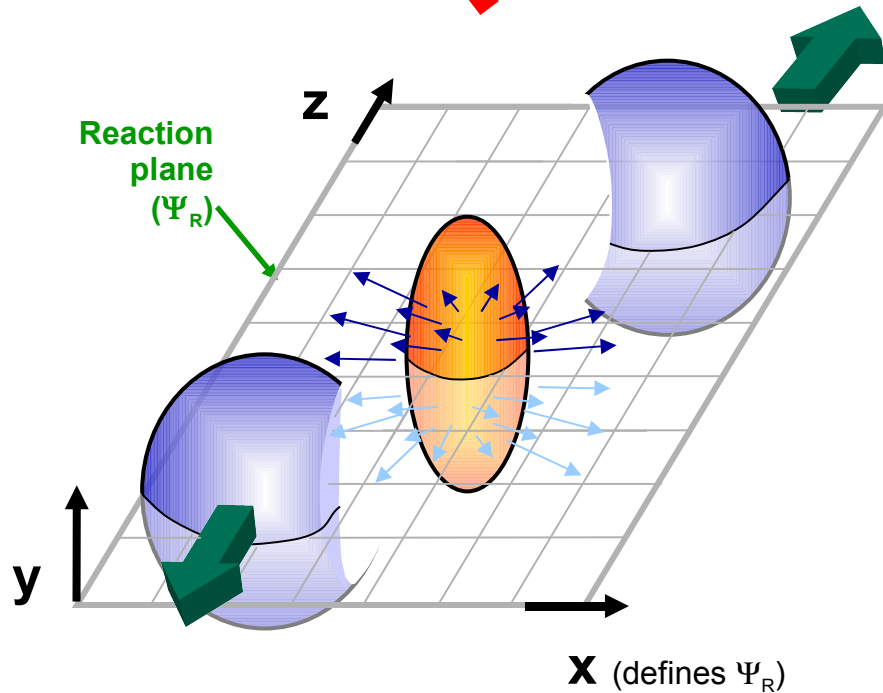
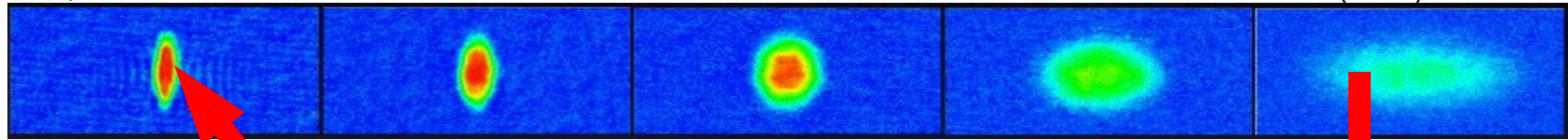
NB: Essentially all talks in our session deal with the second part of the sentence

<http://www.aip.org/pnu/2005/split/757-1.html>
RHIC whitepapers: NPA 757 (2005) 1-283

Initial state anisotropy and elliptic flow

Time →

Illustration from Science 298 5601 (2002) 2179-2182



Initial spatial anisotropy
eccentricity ϵ

→
Interactions
present early

Momentum space anisotropy

$$v_2 = \langle \cos(2\phi - 2\Psi_R) \rangle$$

Elliptic flow and ideal hydro

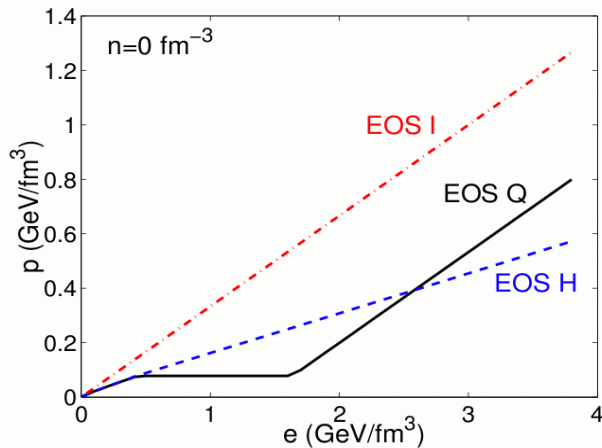
Ideal relativistic hydrodynamics

$$T^{\mu\nu} = (e + p)u^\mu u^\nu - p g^{\mu\nu}$$

$$\delta_\mu T^{\mu\nu} = 0$$

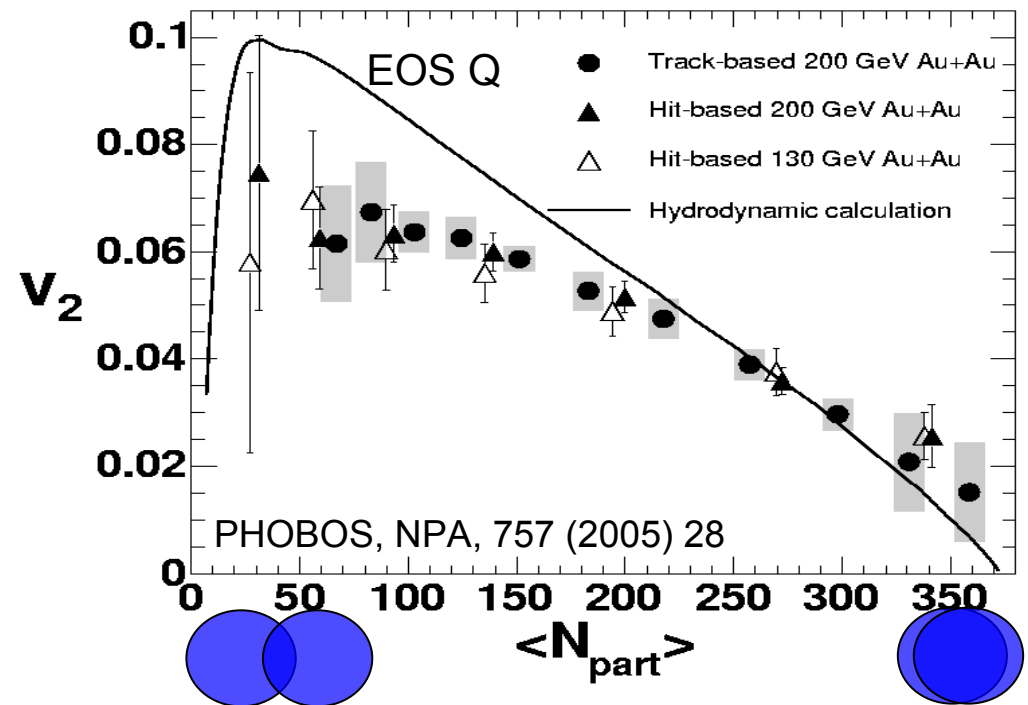
$$\delta_\mu N_i^\mu = 0, \quad i = B, S, \dots$$

$$p = p(e, n) \quad \text{Closure with EoS}$$



Assumption: Shortly after the initial collision ($<1-2$ fm/c) a system in **local equilibrium** with zero mean free path and zero viscosity is created

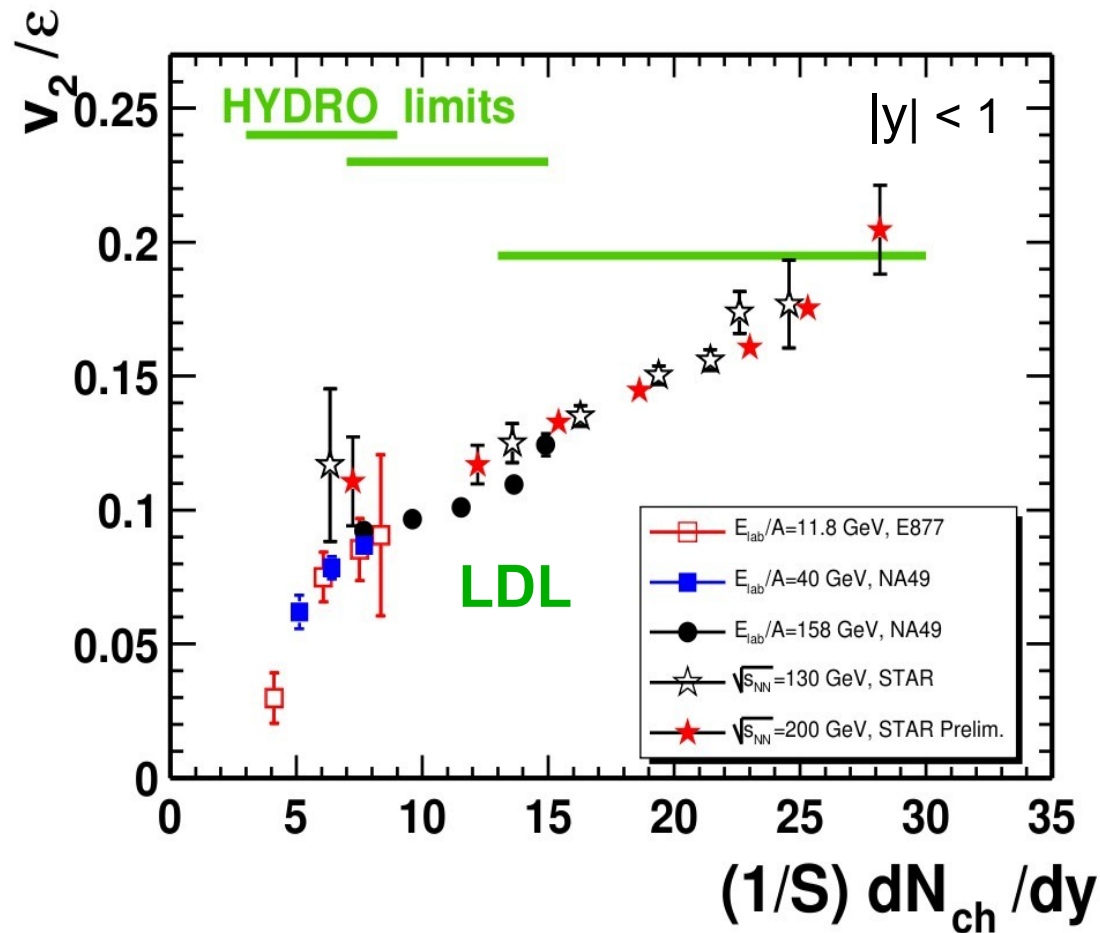
Initial conditions →
Equation of state → **Hydro** → Observables
Freeze-out conditions →



For the first time in history of HI collisions: Mid-central data (200 GeV Au+Au) reach hydro prediction!!!

For complete set of results, see RHIC whitepapers: NPA 757 (2005) 1-283

Eccentricity scaling

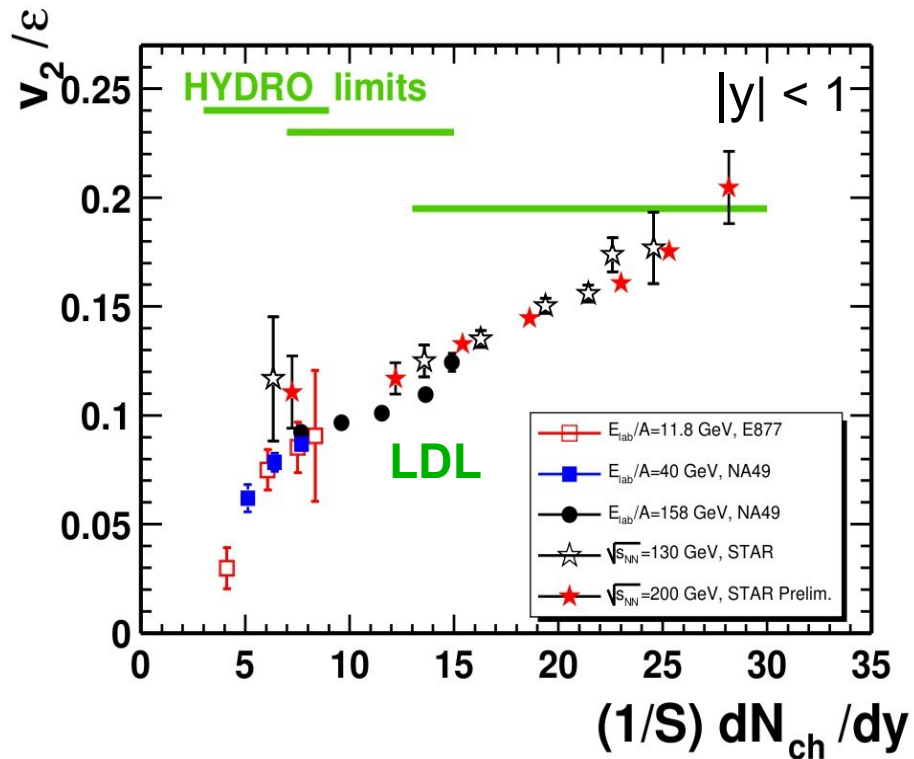


Geometry cancels out in the v_2/ϵ ratio.
In the low density limit region, the ratio rises with the number of collisions per particle, until it saturates in the hydro limit.

Heiselberg, Levy, PRC 59 (1999) 2716
Voloshin, Poskanzer, PLB 474 (2000) 27
STAR, PRC 66 034904 (2002)
NA49, PRC 68 (2003) 034903

“... something more like a liquid ...”

There are many components in this picture: We have and we still are questioning each of them

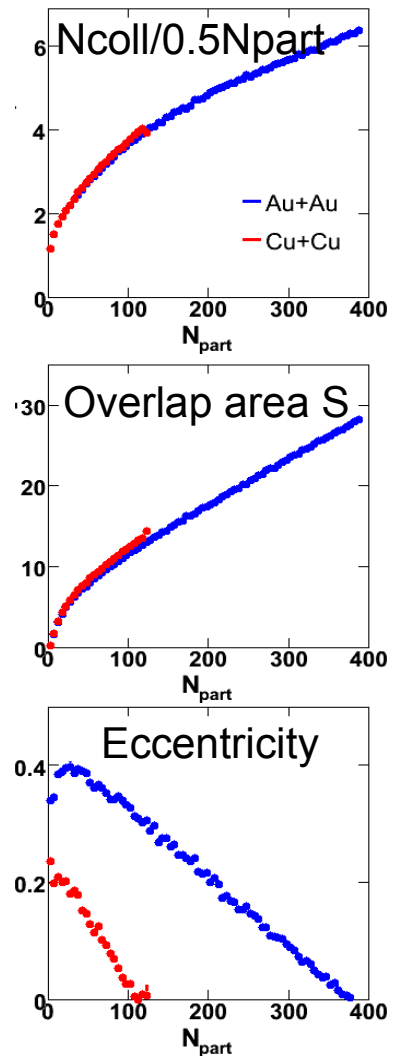
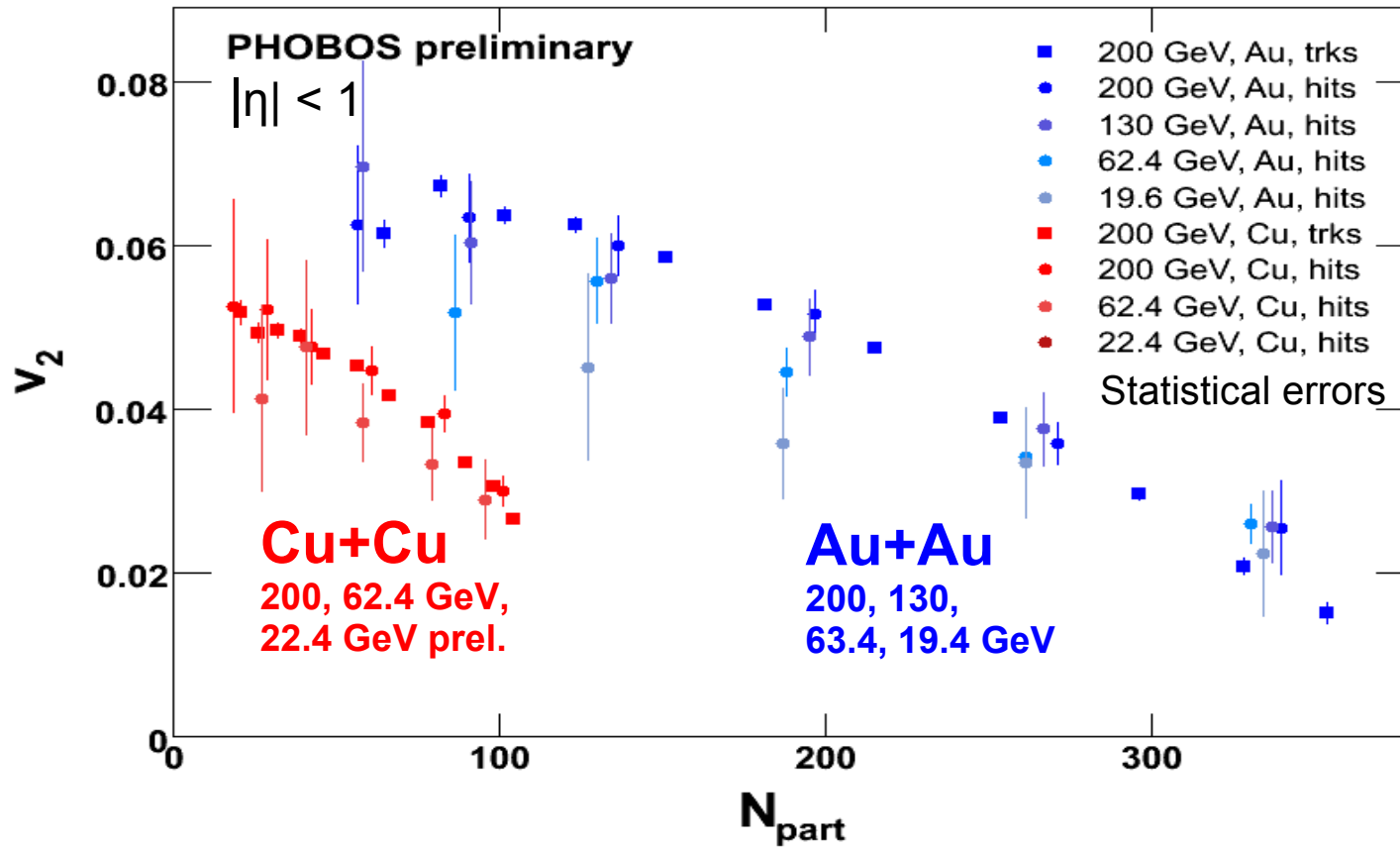


Note:

“Data points” in this plot depend on initial state model for ϵ and S .

- How well do we measure and understand flow
 - System comparison
 - Flow and non-flow fluctuations
 - Sensitivity of mean flow methods to underlying fluctuations
- How well do we model the initial conditions
 - Definition of eccentricity
 - Glauber vs CGC model
- What is the role of the viscosity
 - See talk of Huichao Song
- Is there saturation?

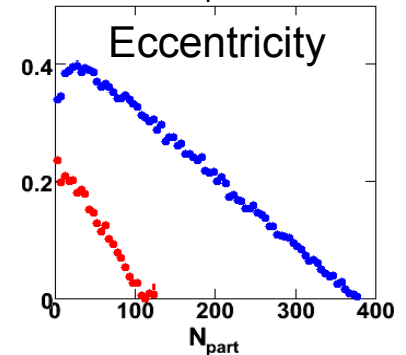
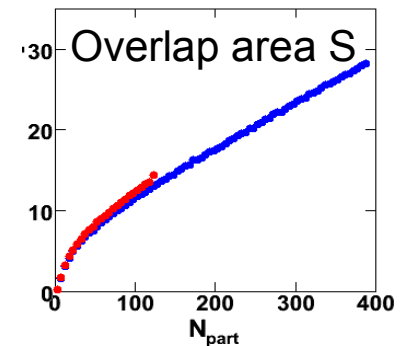
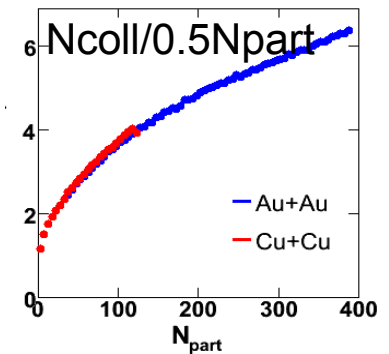
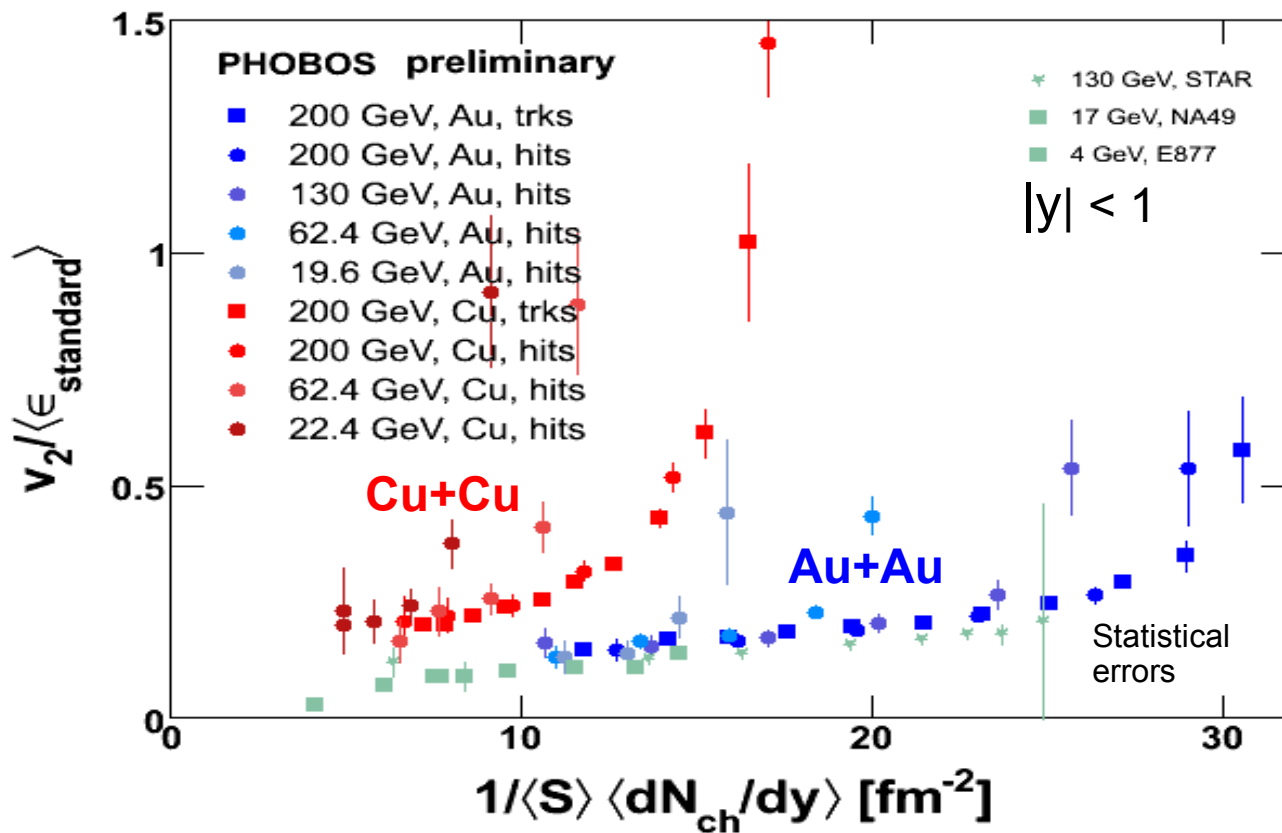
Elliptic flow and collision geometry



Expect eccentricity scaling
 between Cu+Cu and Au+Au

Au+Au, 200,130,62.4+19.6 GeV: PRL 94 (2005) 122303
 Cu+Cu, 200+62.4 GeV: PRL 98 (2007) 242302
 Cu+Cu, 22.4 GeV: prel. QM06
 STAR+NA49+E877, PRC 66 (2002) 034904

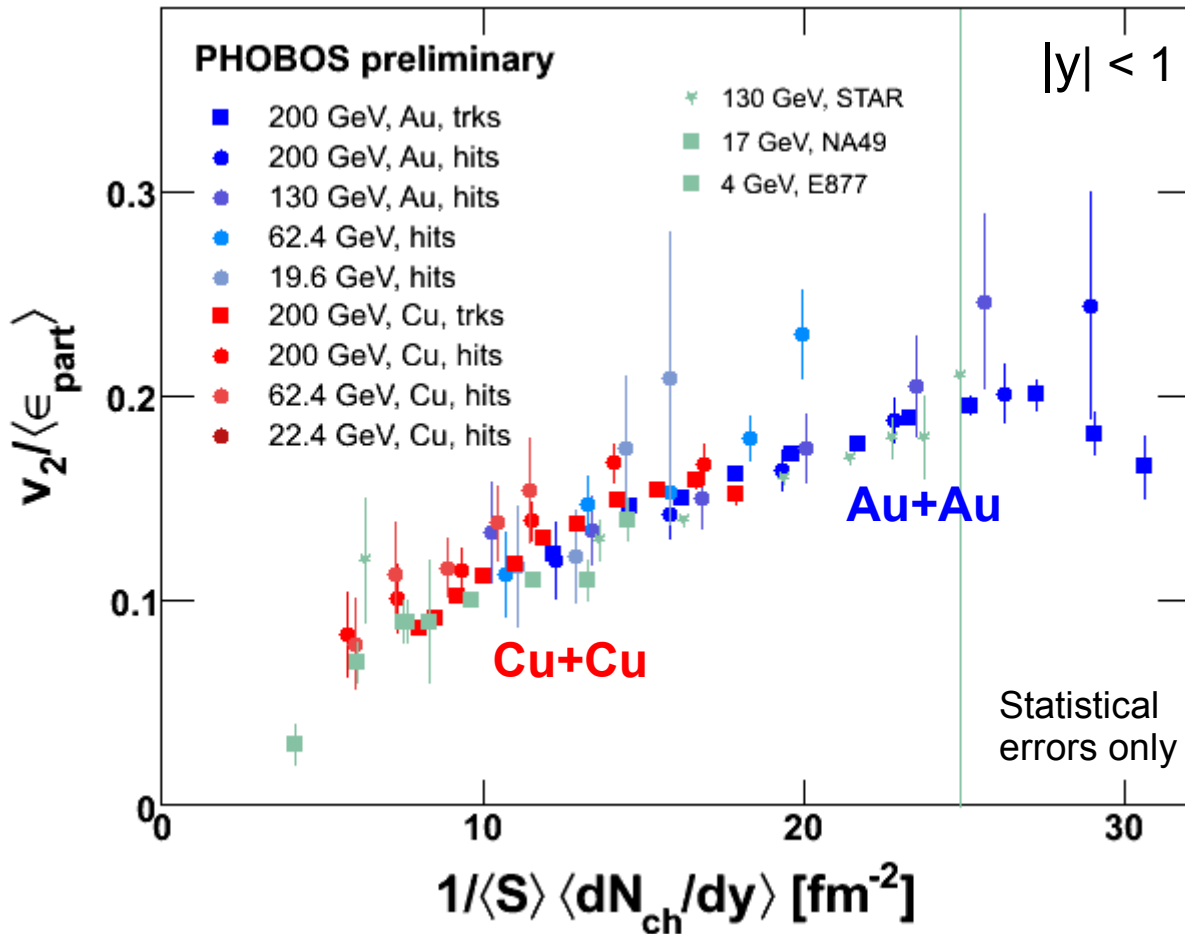
Elliptic flow and collision geometry (2)



No eccentricity scaling
between **Cu+Cu** and **Au+Au**

Au+Au, 200,130,62.4+19.6 GeV: PRL 94 (2005) 122303
 Cu+Cu, 200+62.4 GeV: PRL 98 (2007) 242302
 Cu+Cu, 22.4 GeV: prel. QM06
 STAR+NA49+E877, PRC 66 (2002) 034904

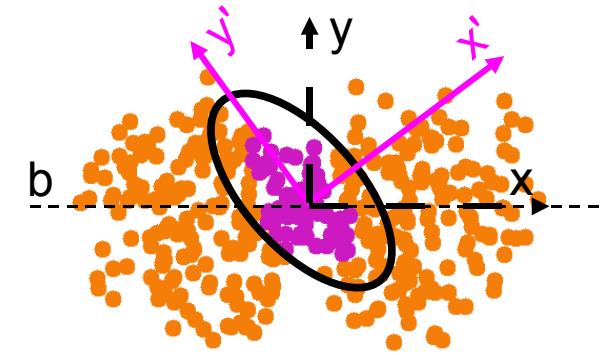
Participant eccentricity scaling



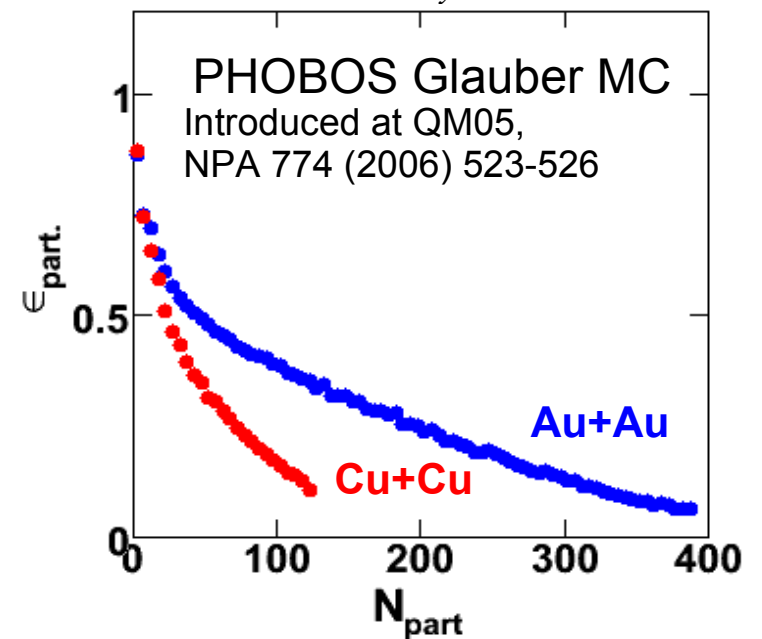
Scaling between **Cu+Cu** and **Au+Au**

Au+Au, 200,130,62.4+19.6 GeV: PRL 94 (2005) 122303
 Cu+Cu, 200+62.4 GeV: PRL 98 (2007) 242302
 Cu+Cu, 22.4 GeV: prel. QM06
 STAR+NA49+E877, PRC 66 (2002) 034904

Participant Eccentricity



$$\epsilon_{part} = \frac{\sqrt{(\sigma_y^2 - \sigma_x^2)^2 + 4\sigma_{xy}^2}}{\sigma_y^2 + \sigma_x^2}$$



Monte Carlo Glauber (MCG) calculation

- Makeup of nuclei

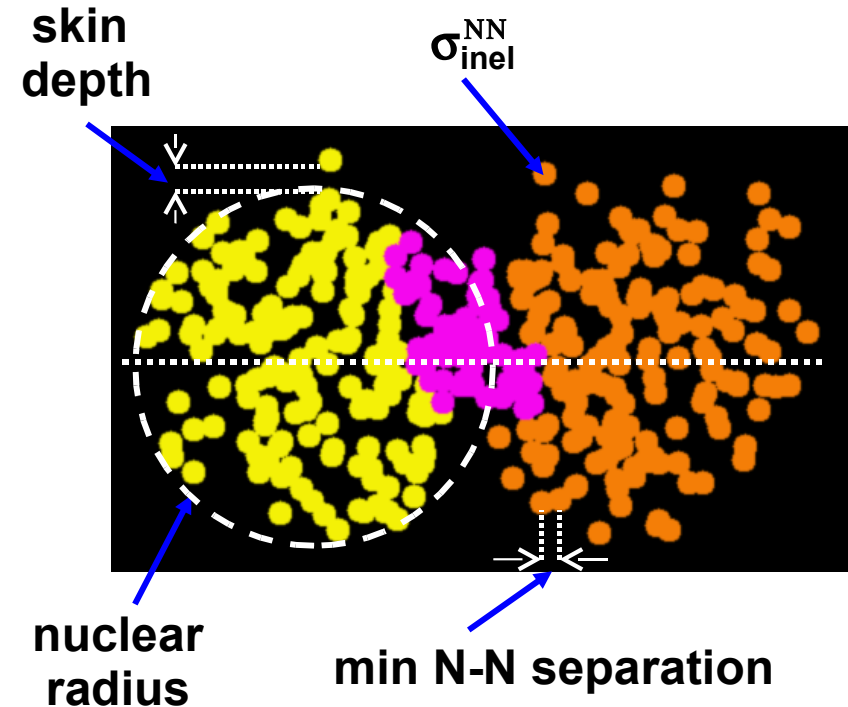
- Radial distribution of nucleons (in nucleus) drawn from Wood-Saxon distribution (given by skin depth, radius)
- Isotropic angular distribution
- Nucleons in nucleus have to fulfill min N-N separation distance

- Simulation of collision

- Separate two nuclei by b (with $dN/db \sim b$)
- Assume: Nucleons travel on straight-line paths and interact inelastically when

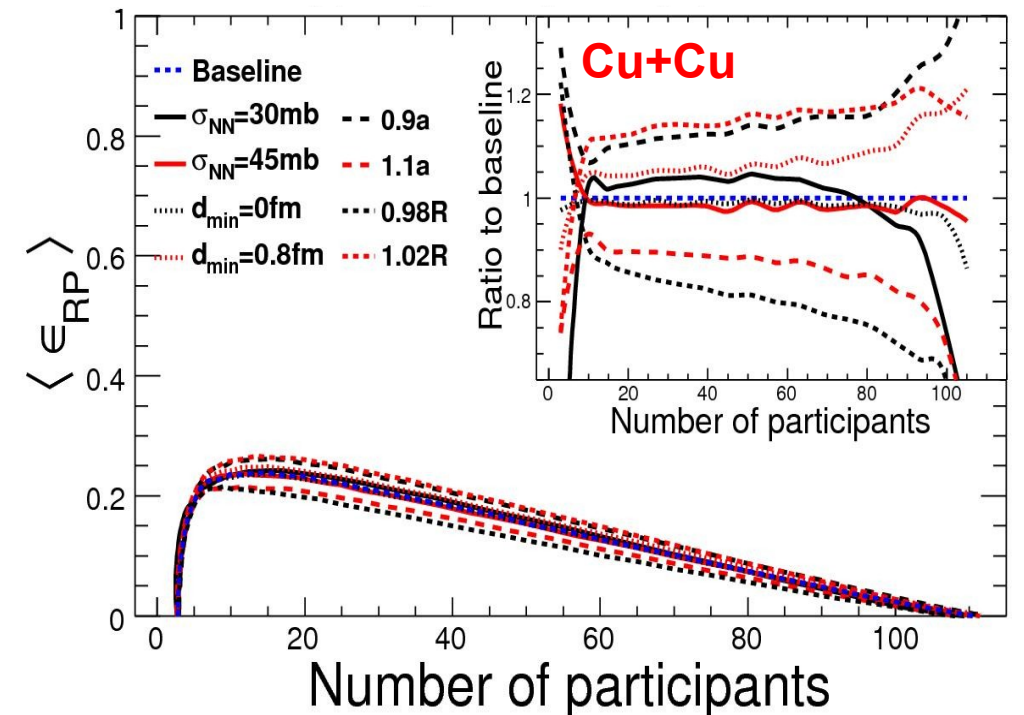
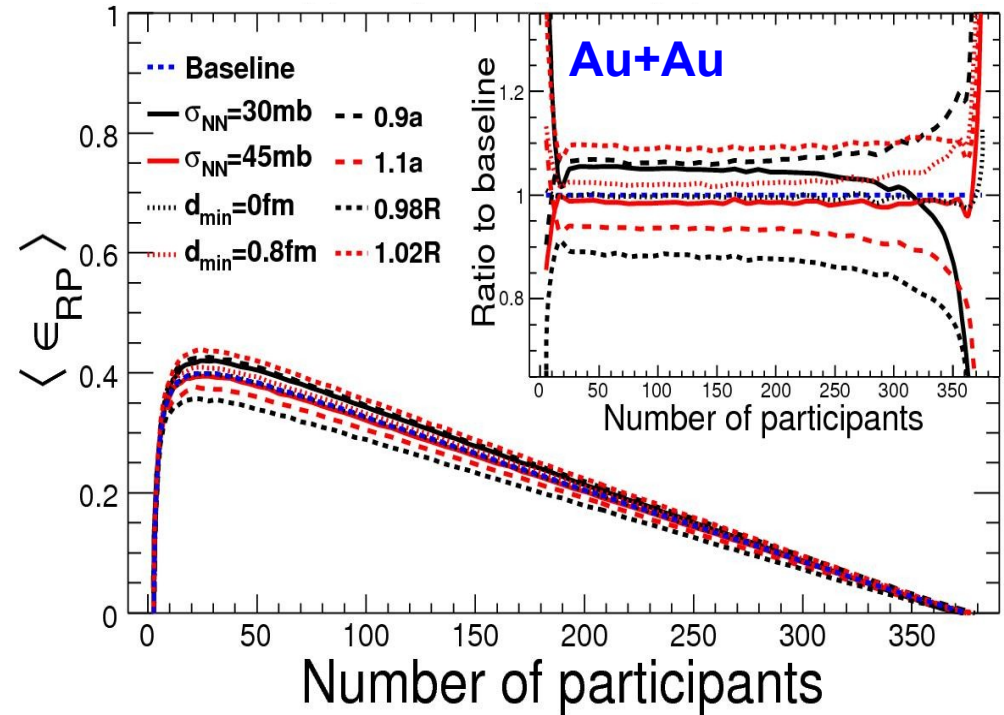
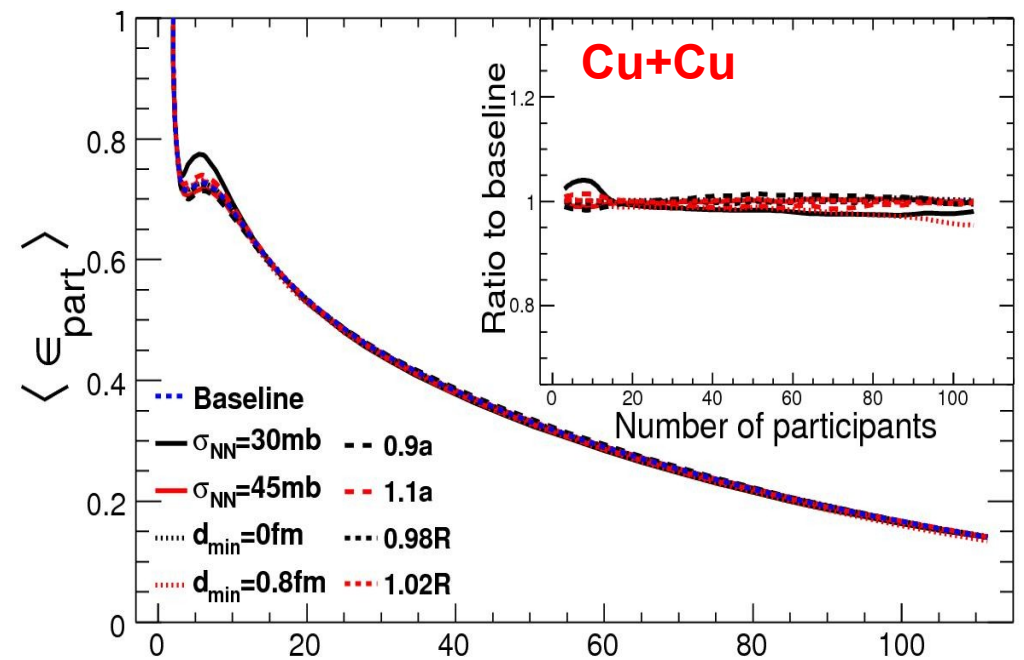
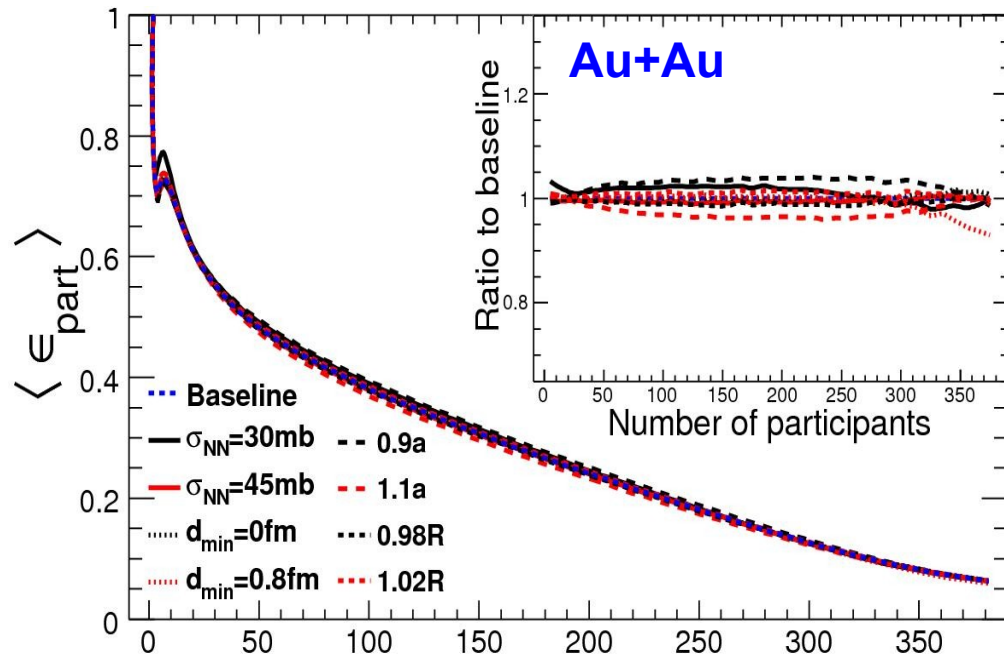
$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} < \sqrt{\sigma_{inel}^{NN} / \pi}$$

- #Participants ($N_{part} \sim A$)
 - Nucleons that interact at least once
- #NN-collisions ($N_{coll} \sim A^{4/3}$)
 - Total number of collisions suffered by the nucleons of one of the nuclei

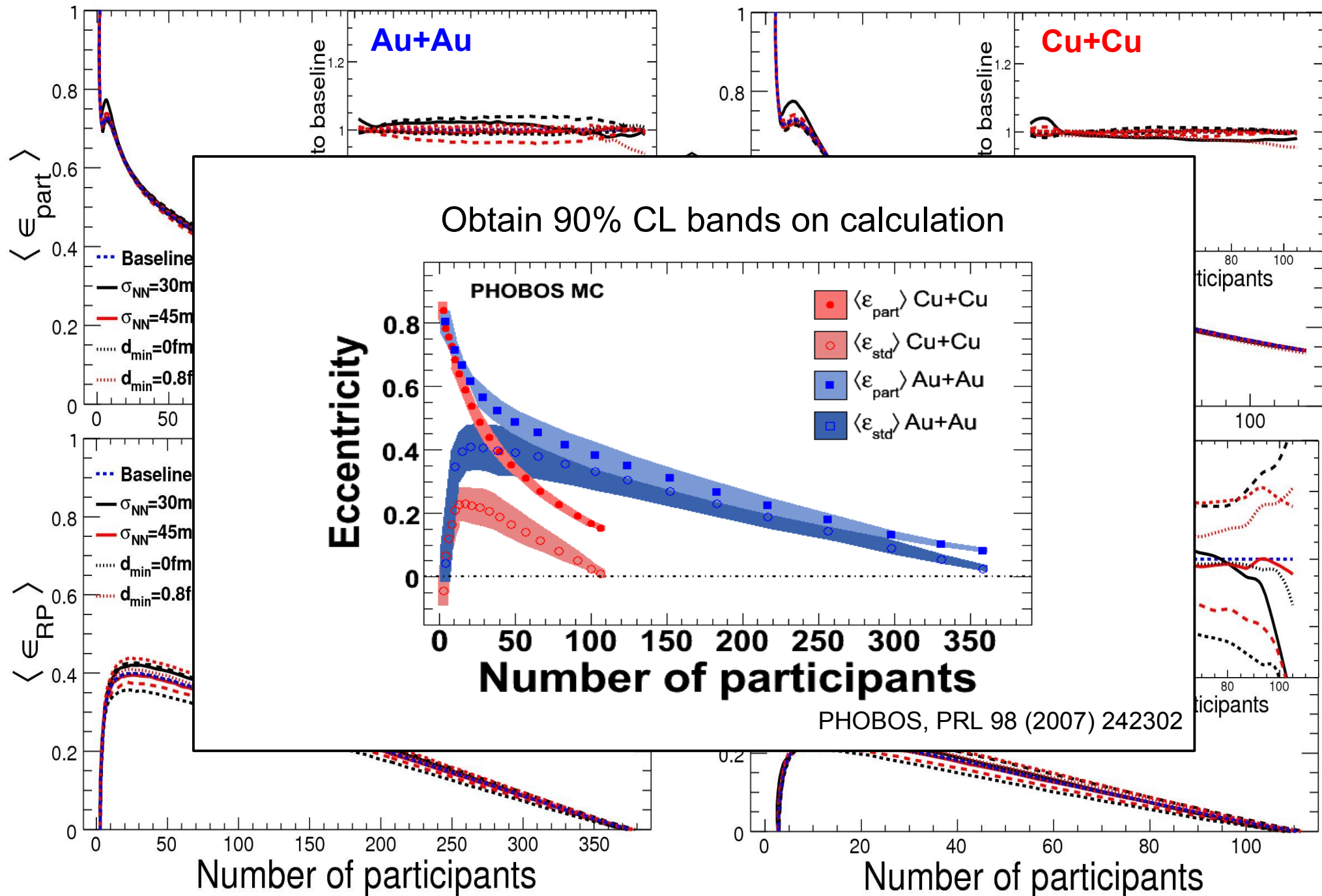


Since MCG was mainly used for average rather than event-by-event quantities, study variations of parameters, and different assumptions about matter production

Variation of eccentricity wrt Glauber parameters



Variation of eccentricity wrt Glauber parameters



Varying assumptions about particle production

- Model two component scenario

- Matter production via participants and binary collisions

$$\frac{dN^{AA}}{d\eta} = \frac{dN^{pp}}{d\eta} \left(\frac{1-x}{2} N_{part} + x N_{coll} \right)$$

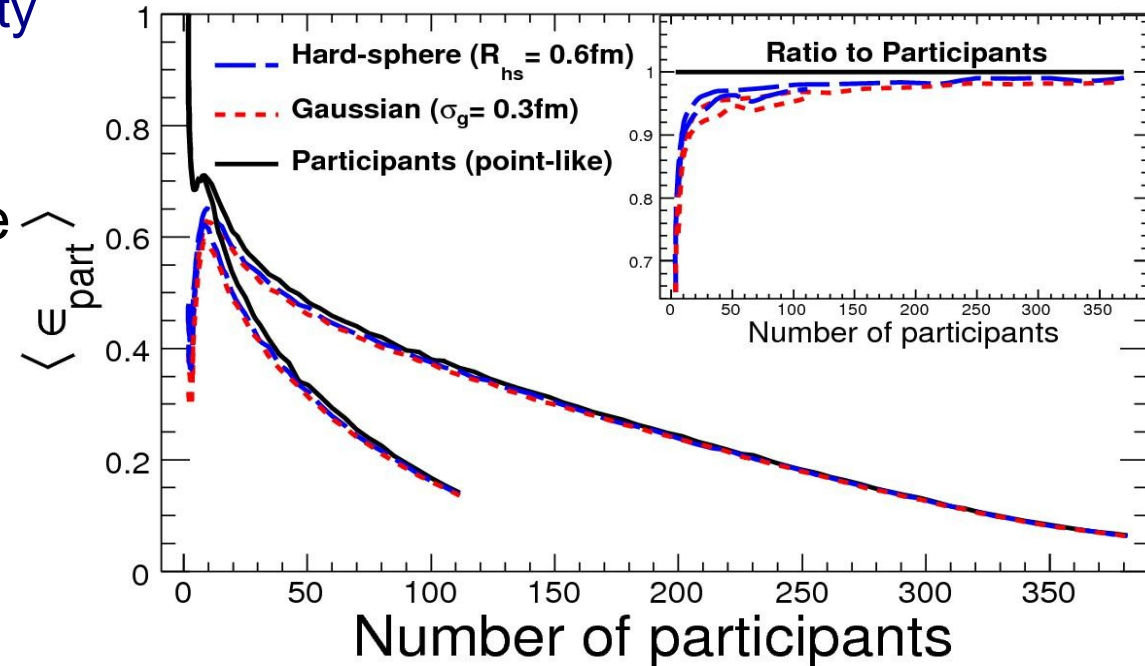
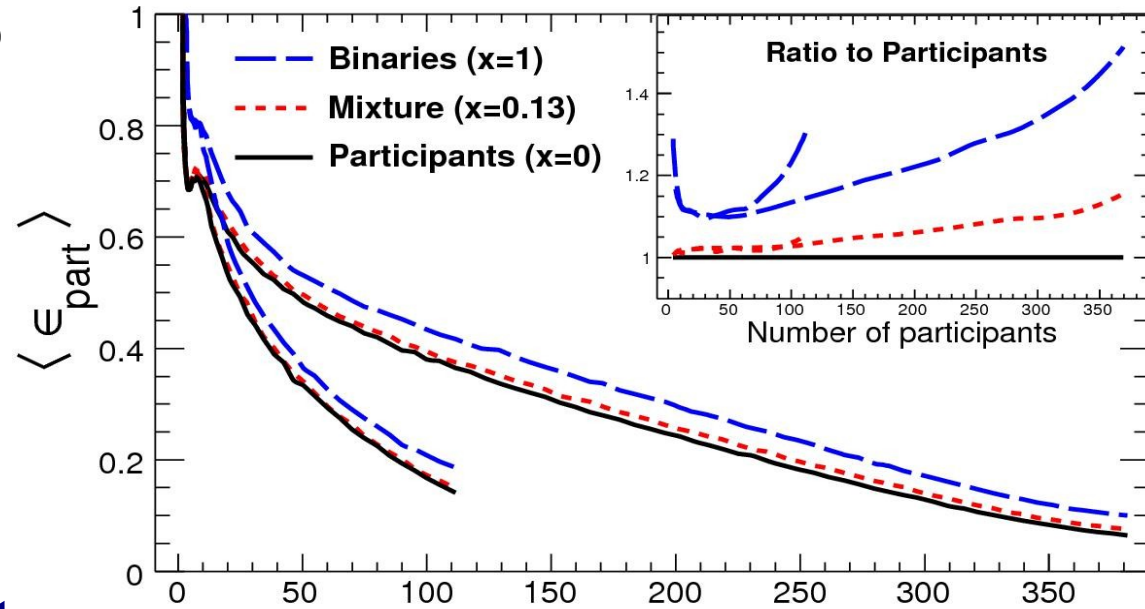
- Mixture with $x=0.13$ describes mid-rapidity $dN/d\eta$ quite well

- 10% increase in eccentricity for central Au+Au

- Include thermalization time by smearing the matter around the original production point

- Hard-sphere and Gaussian

- For chosen set of parameters only a very small effect



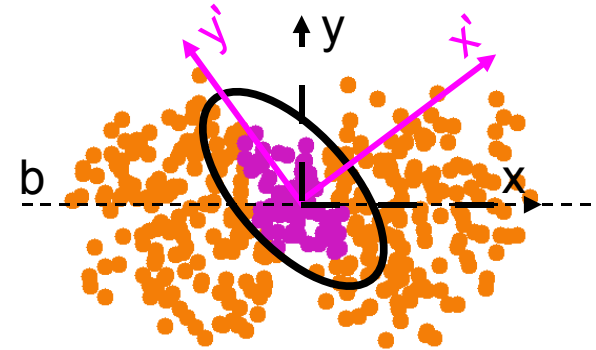
NB: More generalized studies also done, see Broniowski et al., PRC 76 (2007) 054905

Alver et al., PRC 77 (2008) 014906

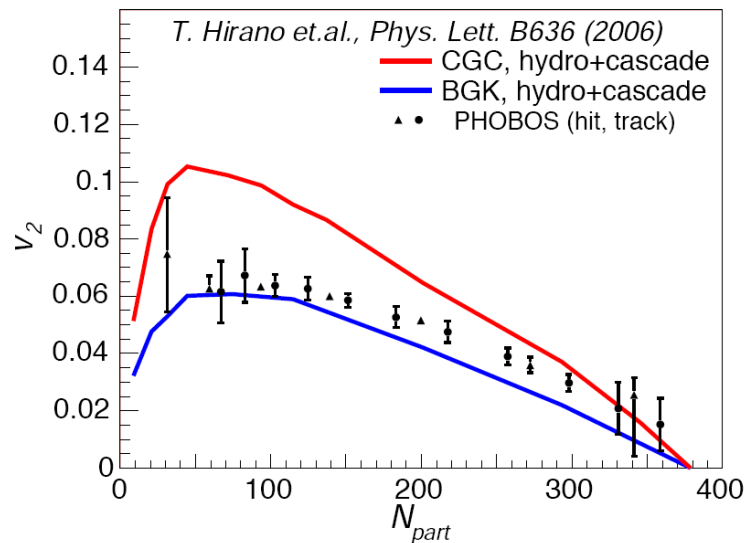
Measuring v_2 fluctuations

Elliptic flow is developed **event-by-event** with respect to the overlap region, expect

$$\frac{\sigma_{v_2}}{\langle v_2 \rangle} \sim \frac{\sigma_{\epsilon_{part}}}{\langle \epsilon_{part} \rangle}$$



Motivation

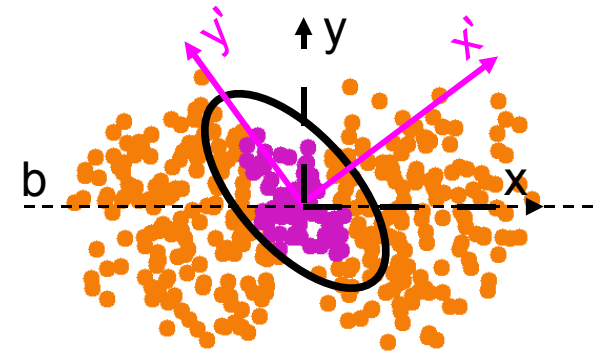


- CGC initial conditions yield larger eccentricity than Glauber based ones
- CGC initial conditions leave room for viscous corrections to ideal hydro, since higher eccentricity would imply higher flow
- Flow fluctuations may be an observable sensitive to initial conditions

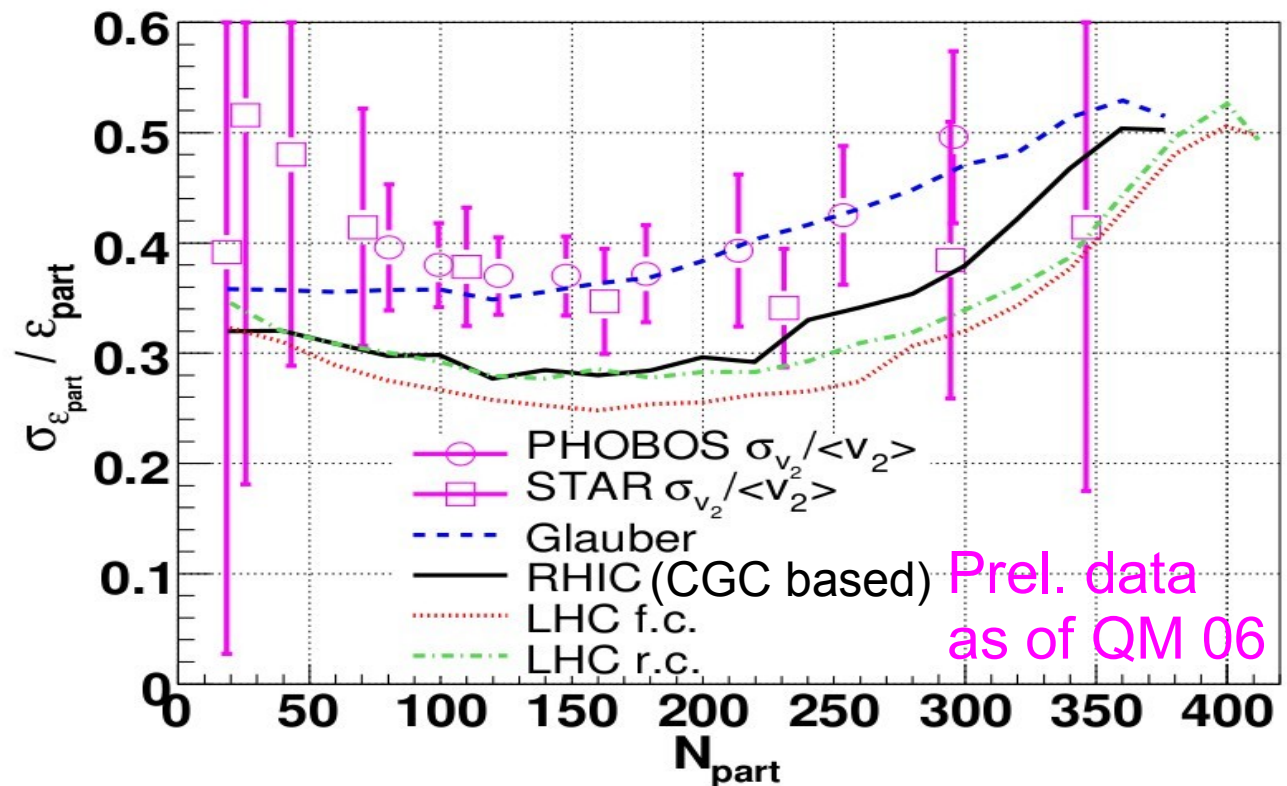
Measuring v_2 fluctuations

Elliptic flow is developed **event-by-event** with respect to the overlap region, expect

$$\frac{\sigma_{v_2}}{\langle v_2 \rangle} \sim \frac{\sigma_{\epsilon_{part}}}{\langle \epsilon_{part} \rangle}$$



- Preliminary data of QM06 seemed to prefer Glauber based initial conditions
- However, contribution of non-flow correlations not yet settled (see next slides)



Non-flow contribution to v_2 fluctuations

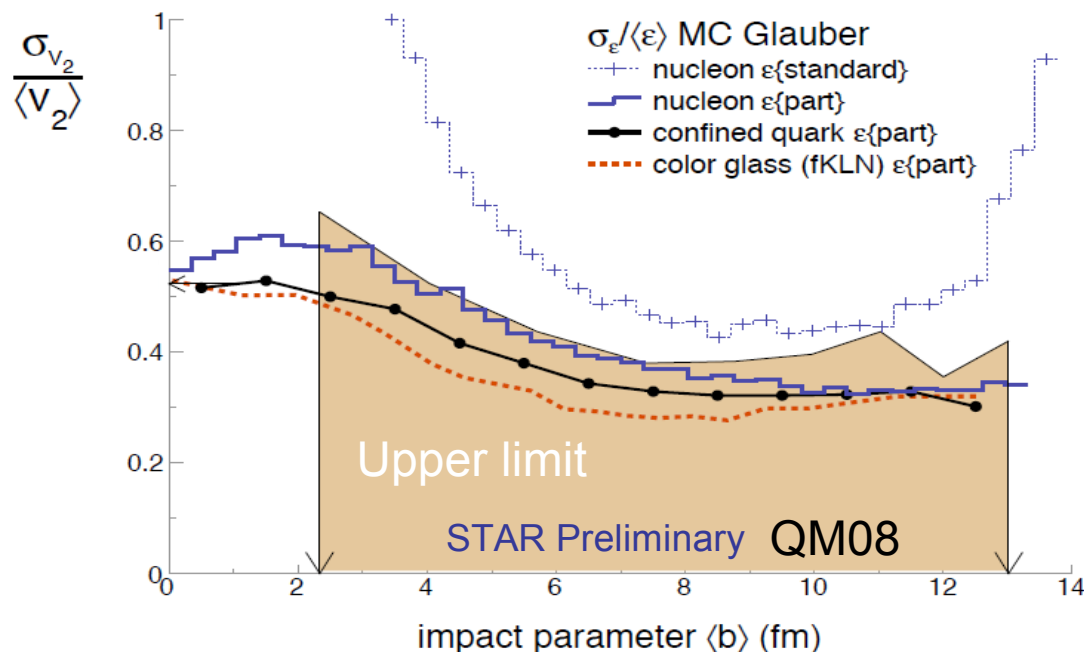
Non-flow correlations are few particle correlations not related to the orientation of Ψ_R

$$\langle v_2^2 \rangle = \langle \cos(2\phi_i - 2\phi_j) \rangle = v_2^2 + \delta$$

Non-flow correlations broaden the observed flow fluctuations in a non-trivial way.

For Gaussian (*) flow fluctuations one can show that the flow (q-vector) distribution is fully described by $v_2\{2\}$ and $v_2\{4\}$, which leads to an ambiguity in the determination of the three unknowns: $\langle v_2 \rangle$, σ_{v_2} and σ_δ

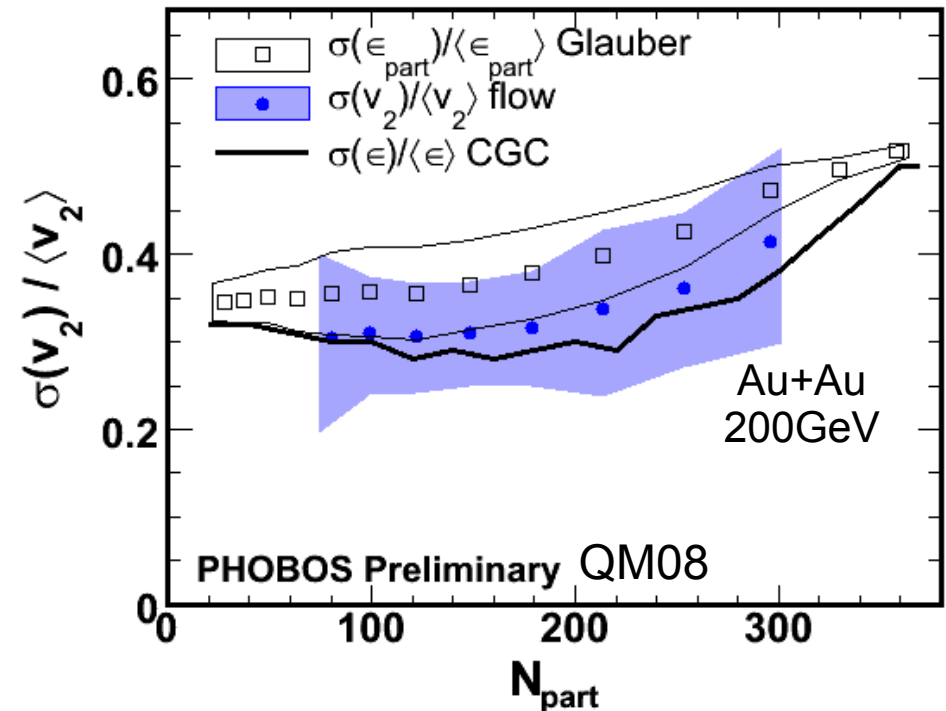
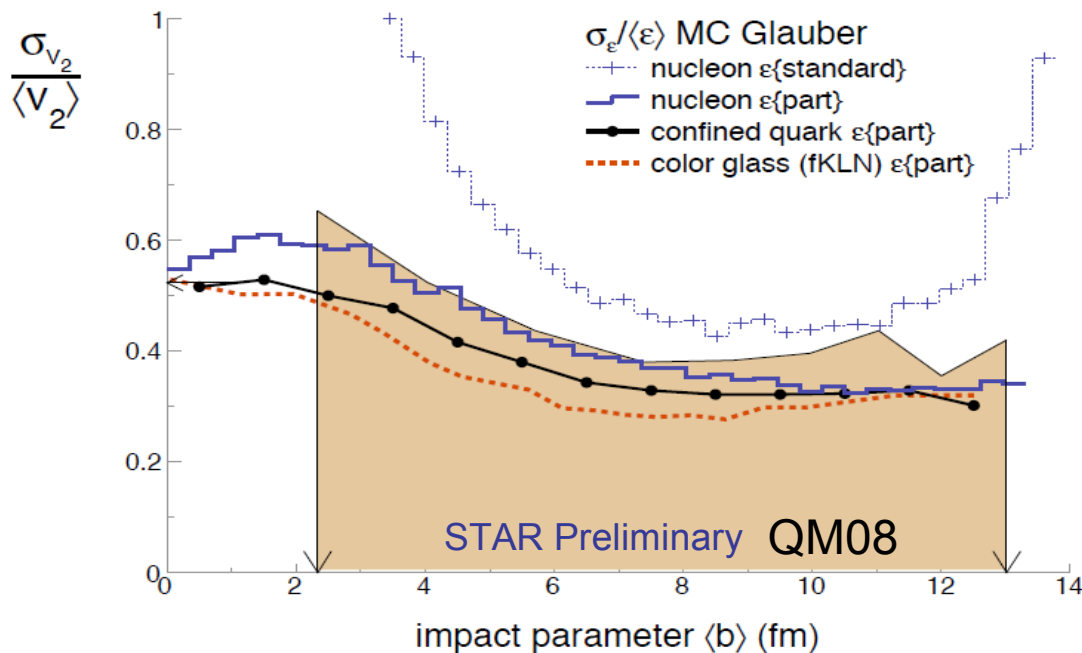
Voloshin et.al., PLB 659 (2008) 537-541



*) For central collisions, eccentricity fluctuations modeled with Glauber are very well described by Gaussians. For peripheral collisions however the approximation fails.

Non-flow contribution to v_2 fluctuations

- PHOBOS uses data and MC driven analysis to measure flow and non-flow
 - Flow is a function of η and correlates at all $\Delta\eta$ ranges
 - Non-flow is dominated by short range correlations (small $\Delta\eta$)
 - Study $\Delta\phi$ correlations at different $\Delta\eta$ ranges
- Assume that HIJING describes shape (not magnitude) of non-flow at $\Delta\eta > 2$
- Connect non-flow to flow fluctuations with a generalized cluster decay MC model



Eccentricity scaling and eccentricity cumulants

- Ideally one would take

$$\left\langle \frac{v_2}{\epsilon} \right\rangle$$

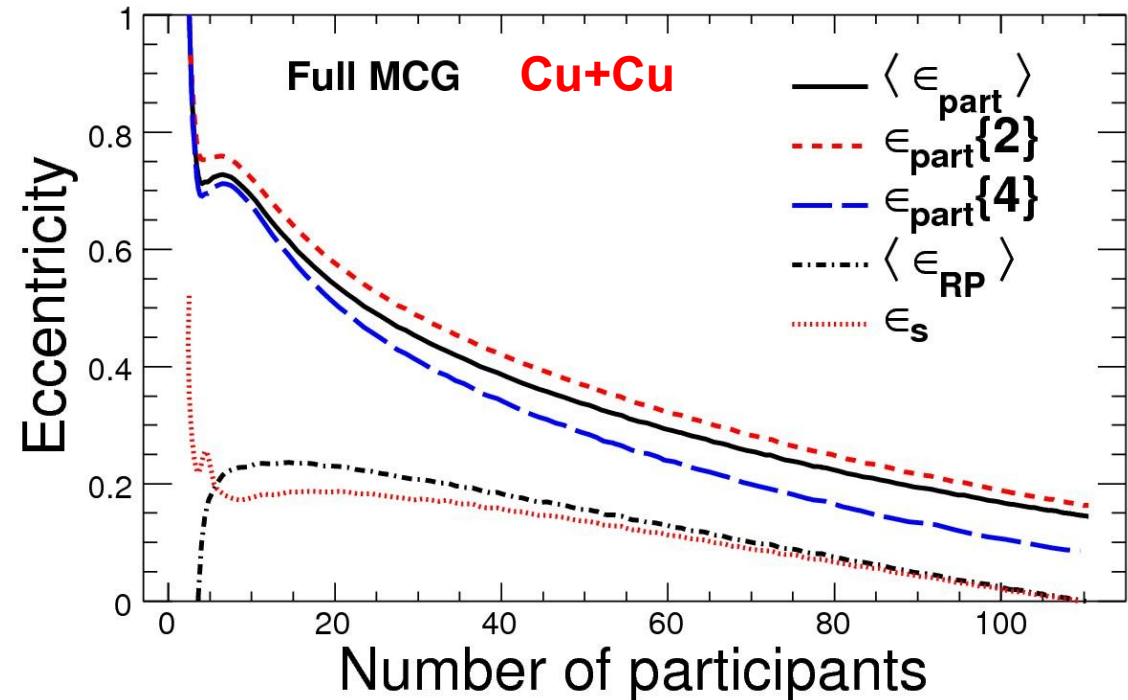
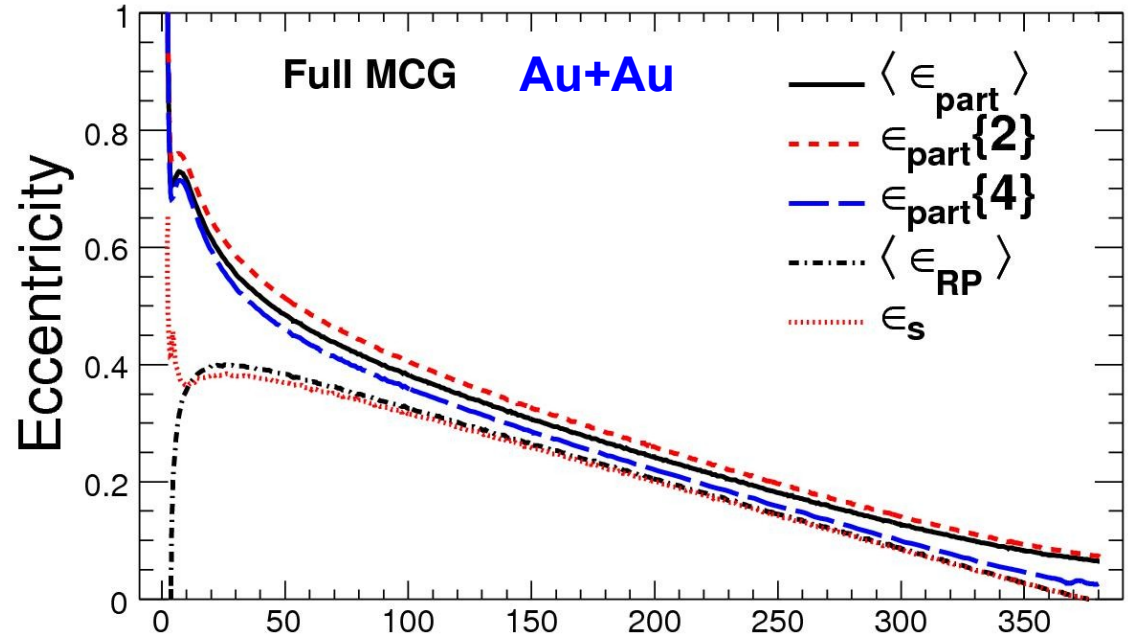
- Since this is not possible, take

$$\frac{\langle v_2 \rangle}{\langle \epsilon \rangle}$$

- Or the corresponding moment and eccentricity definition measured by the flow method, e.g.

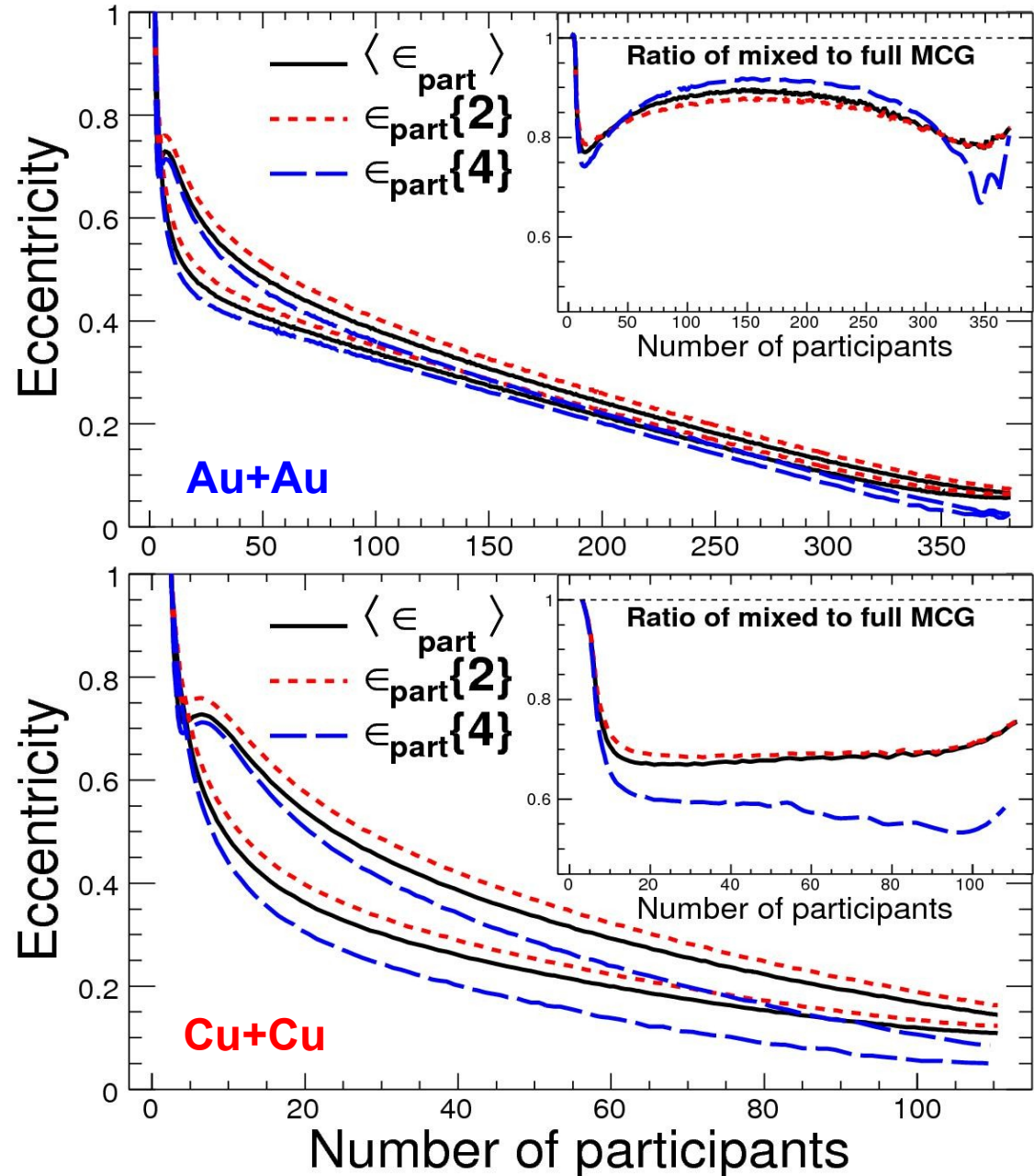
$$\frac{v_2\{2\}}{\epsilon_{part}\{2\}} \text{ or } \frac{v_2\{4\}}{\epsilon_{part}\{4\}} \text{ or } \frac{v_2\{ZDC\}}{\langle \epsilon_{RP} \rangle}$$

Miller, Snellings, nucl-ex/0312008
 Bhalerao, Ollitrault, PLB 641 (2006) 260-264



Calculating eccentricity cumulants

- Calculations using optical models neglect inter-NN and inter-participant correlations *)
- Study importance of correlations among participants using MC Glauber with mixed events
 - Eccentricity cumulants from mixed events yield an eccentricity of between 10% (Au+Au) to 40% (Cu+Cu) smaller wrt full MC calculation
 - Especially for the smaller system, participant correlations must be taken into account



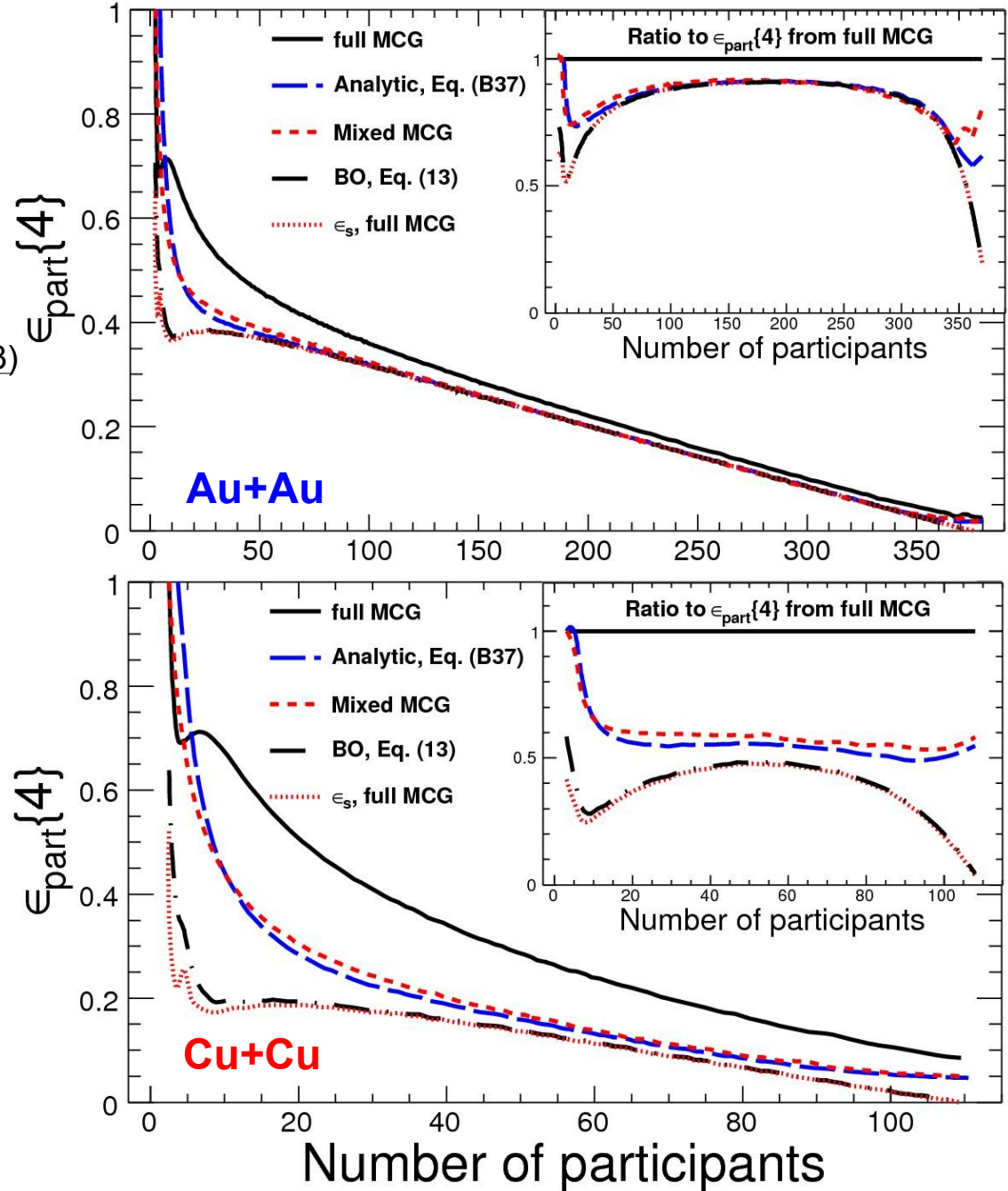
*) All Glauber models ignore genuine NN correlations from wavefunction

Comparing analytical to MC eccentricity cumulants

- Extended expansion of BO to higher orders in $1/N_{\text{part}}$
 - Truncated higher order $1/N_{\text{part}}$ terms are either negligible or included the missing ones

$$\begin{aligned} \epsilon_{\text{part}\{4\}}^4 &= \epsilon_s^4 + \frac{1}{N \langle r^2 \rangle^2} [2\epsilon_s^4 \langle r^4 \rangle - 2\epsilon_s^2 \langle r^4 \cos 4\phi \rangle] \\ &\quad \text{BO, Eq(13)} \\ &+ \frac{1}{N^2} \left[8\epsilon_s^2 \frac{\langle r^6 \rangle}{\langle r^2 \rangle^3} + 4\epsilon_s \frac{\langle r^6 \cos 2\phi \rangle}{\langle r^2 \rangle^3} \right. \\ &\quad \left. - 16\epsilon_s^2 \frac{\langle r^4 \rangle^2}{\langle r^2 \rangle^4} - 16\epsilon_s \frac{\langle r^4 \rangle \langle r^4 \cos 2\phi \rangle}{\langle r^2 \rangle^4} \right] \\ &+ \frac{1}{N^3} \left[\frac{2 \langle r^4 \rangle^2}{\langle r^2 \rangle^4} - \frac{\langle r^8 \rangle}{\langle r^2 \rangle^4} + \frac{8 \langle r^4 \rangle \langle r^6 \rangle}{\langle r^2 \rangle^5} \right. \\ &\quad \left. - \frac{8 \langle r^4 \rangle^3}{\langle r^2 \rangle^6} \right] + \mathcal{O}\left(\frac{\epsilon_s^4}{N^2}\right) + \mathcal{O}\left(\frac{\epsilon_s^2}{N^3}\right) \\ &+ \mathcal{O}\left(\frac{\epsilon_s^0}{N^4}\right) + \mathcal{O}\left(\frac{\epsilon_s^0}{N^5}\right) + \dots, \quad (\text{B37}) \end{aligned}$$

- Theoreticians should use MC Glauber rather than semi-analytical formulas, if possible

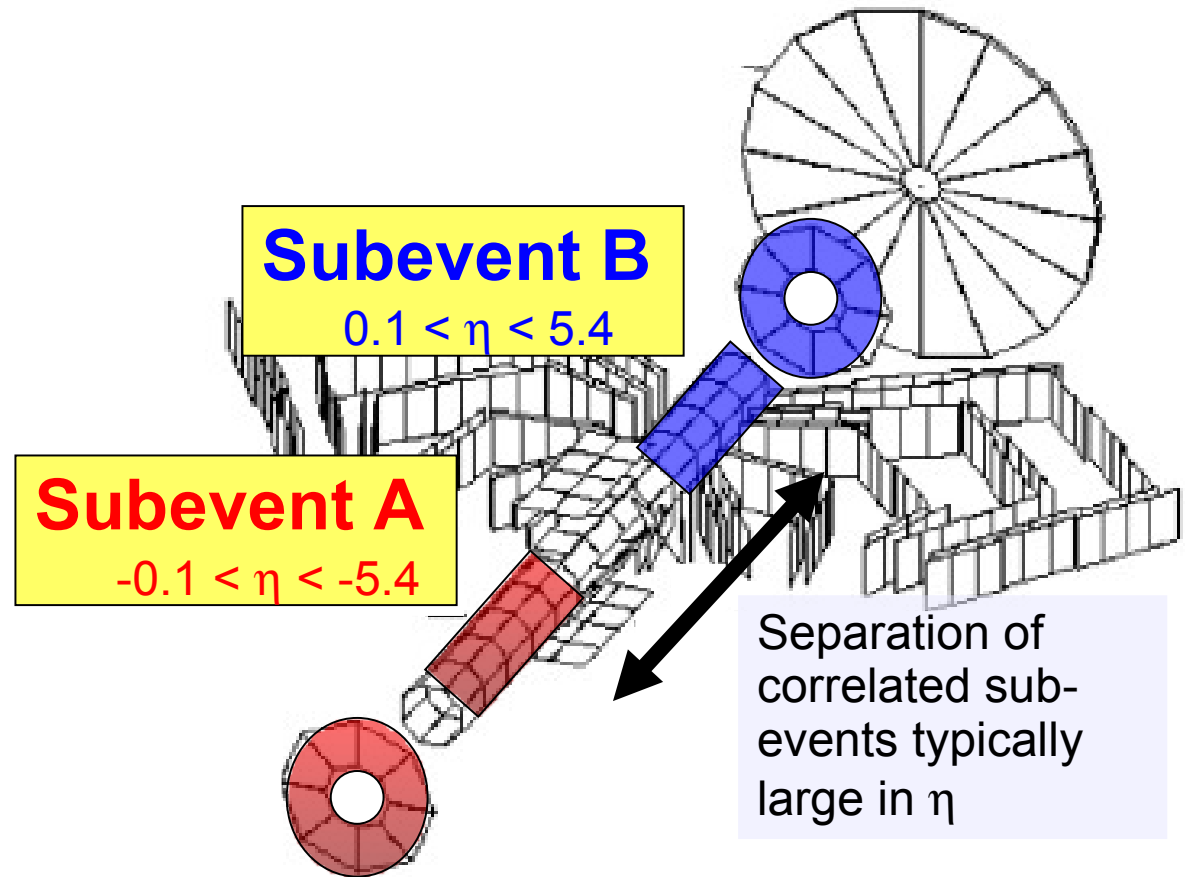


Standard event-plane flow method in PHOBOS

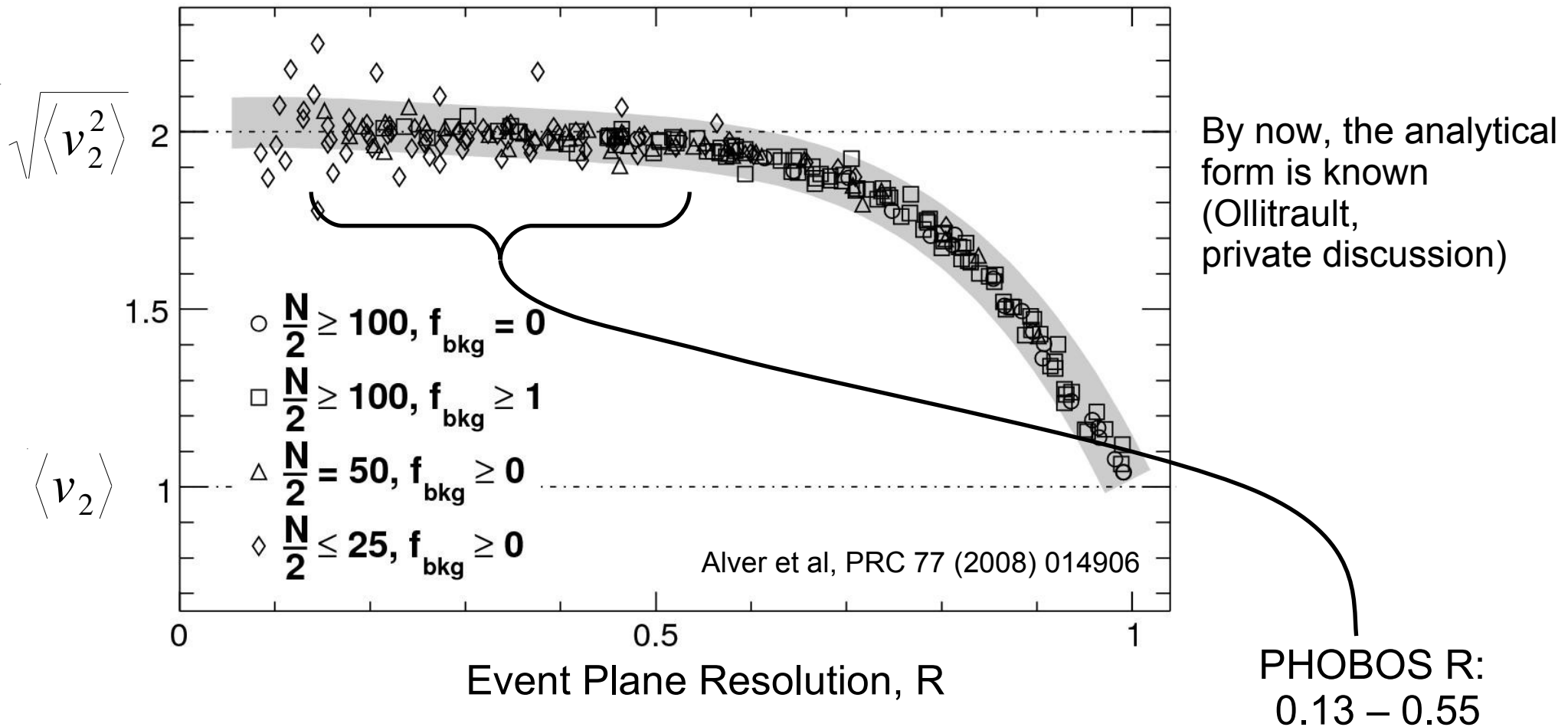
- Reaction-plane / Sub-event technique
 - Correlate reaction plane determined from azimuthal pattern of hits in one part of the detector with information from other parts of the detector

$$\tan(2\psi_A) = \frac{\langle \sin(2\phi) \rangle_A}{\langle \cos(2\phi) \rangle_A}$$
$$V_2^{obs} = \langle \cos(2\phi - 2\psi_A) \rangle_B$$
$$V_2 = \frac{\langle V_2^{obs} \rangle_{events}}{\sqrt{\langle \cos(2\psi_A - 2\psi_B) \rangle_{events}}}$$

A.Poskanzer, S.Voloshin,
nucl-ex/9805001

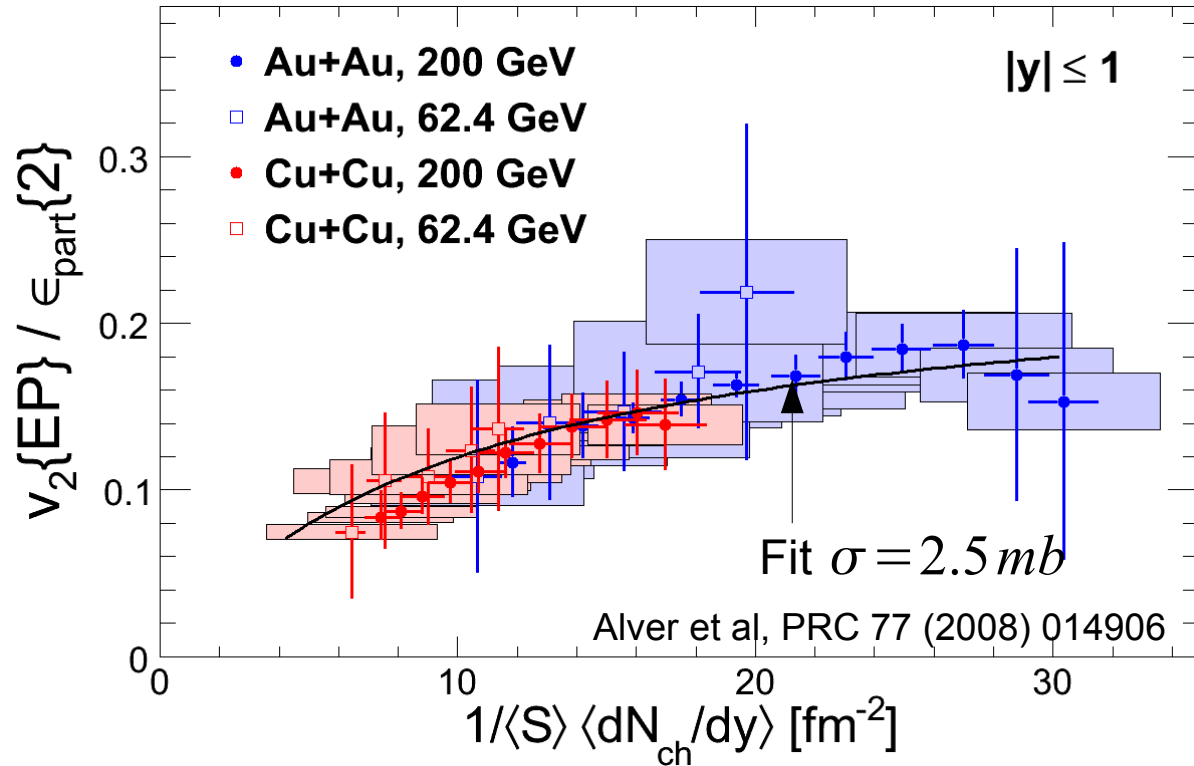


What does standard event-plane method measure?



For PHOBOS standard event-plane method $v_2 \{ EP \} = \sqrt{\langle v_2^2 \rangle}$

How in-complete is the thermalization?



Central Au+Au are about 30% away from ideal hydro limit

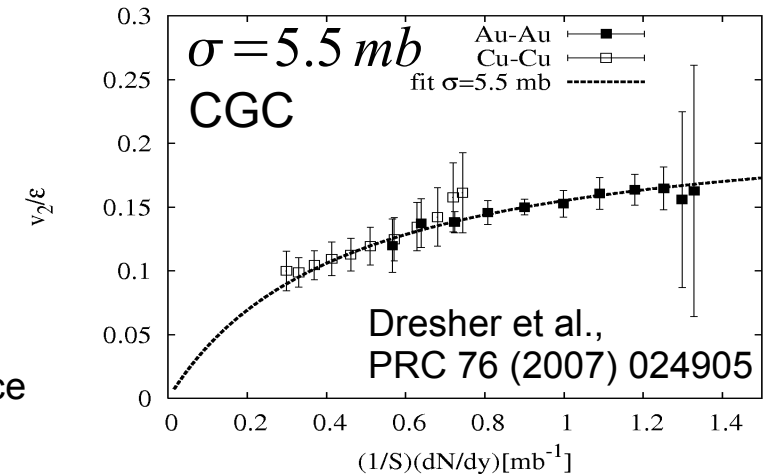
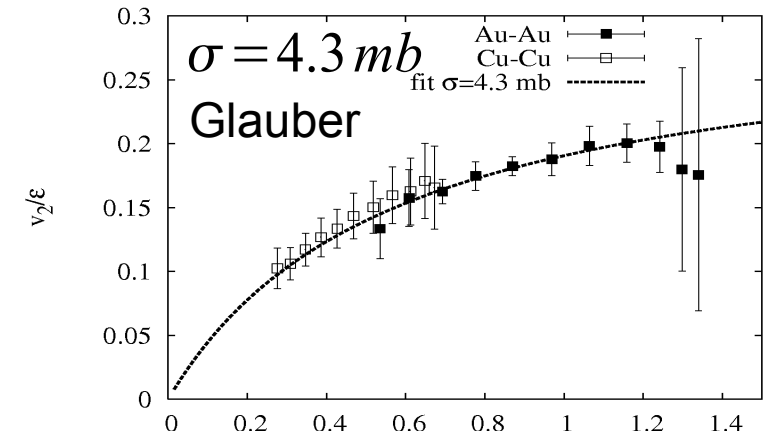
*) Assumed constant c_s , no phase transitions, 2d, boost invariance

**) Difference of factor 2 in horizontal scales

$$\frac{v_2}{\epsilon} = \frac{v_2^{hydro}}{\epsilon} \frac{1}{1 + K/K_0} \quad K_0 \approx 0.7$$

$$\frac{1}{K} = \frac{\sigma}{S} \frac{dN}{dy} \frac{c_s}{c}$$

Bhalerao et al., PLB 627 (2005) 49-54



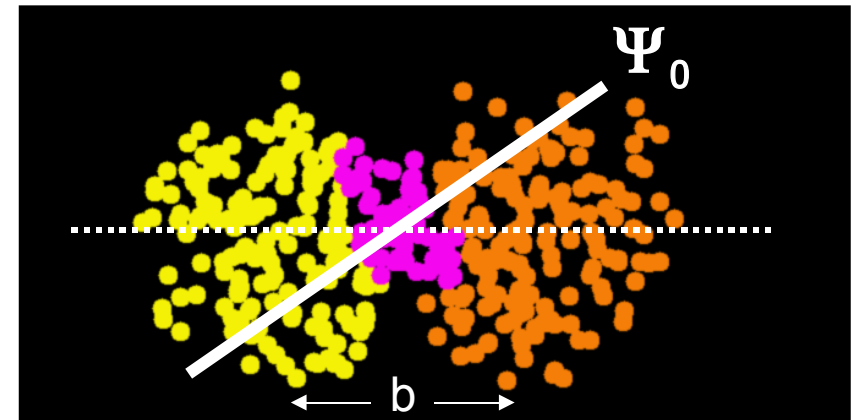
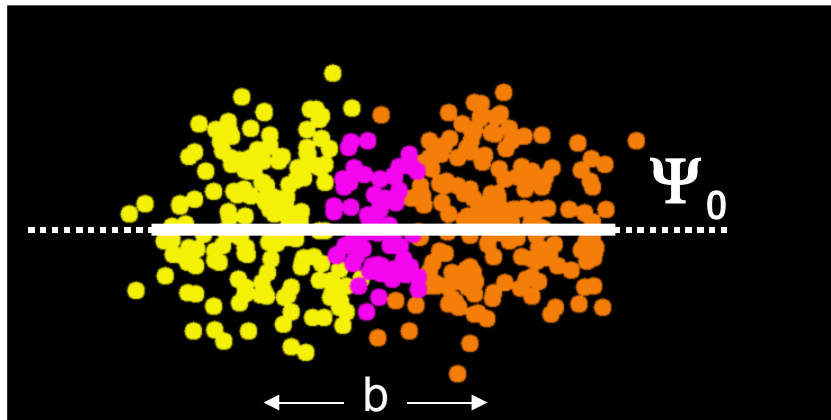
Summary

- Significant progress in understanding and quantifying the “Something more like a liquid”
 - Initial conditions
 - Participant eccentricity unifies across systems
 - Participant eccentricity from MCG robust
 - Participant correlations important for eccentricity cumulants
 - Use of MCG preferred over analytical formulas
 - Flow fluctuations
 - May have potential to discriminate between initial conditions
 - Still work needed to nail down non-flow contributions
 - Standard $v_2\{EP\}$ measures first or second moment dep. on resolution
 - Eccentricity scaling
 - Central Au+Au collisions are 30% away from hydro limit
 - Room for viscous corrections

Backups

Participant eccentricity definition

The spatial distribution of
the interaction points of participating nucleons
for the same b will vary from event-to-event



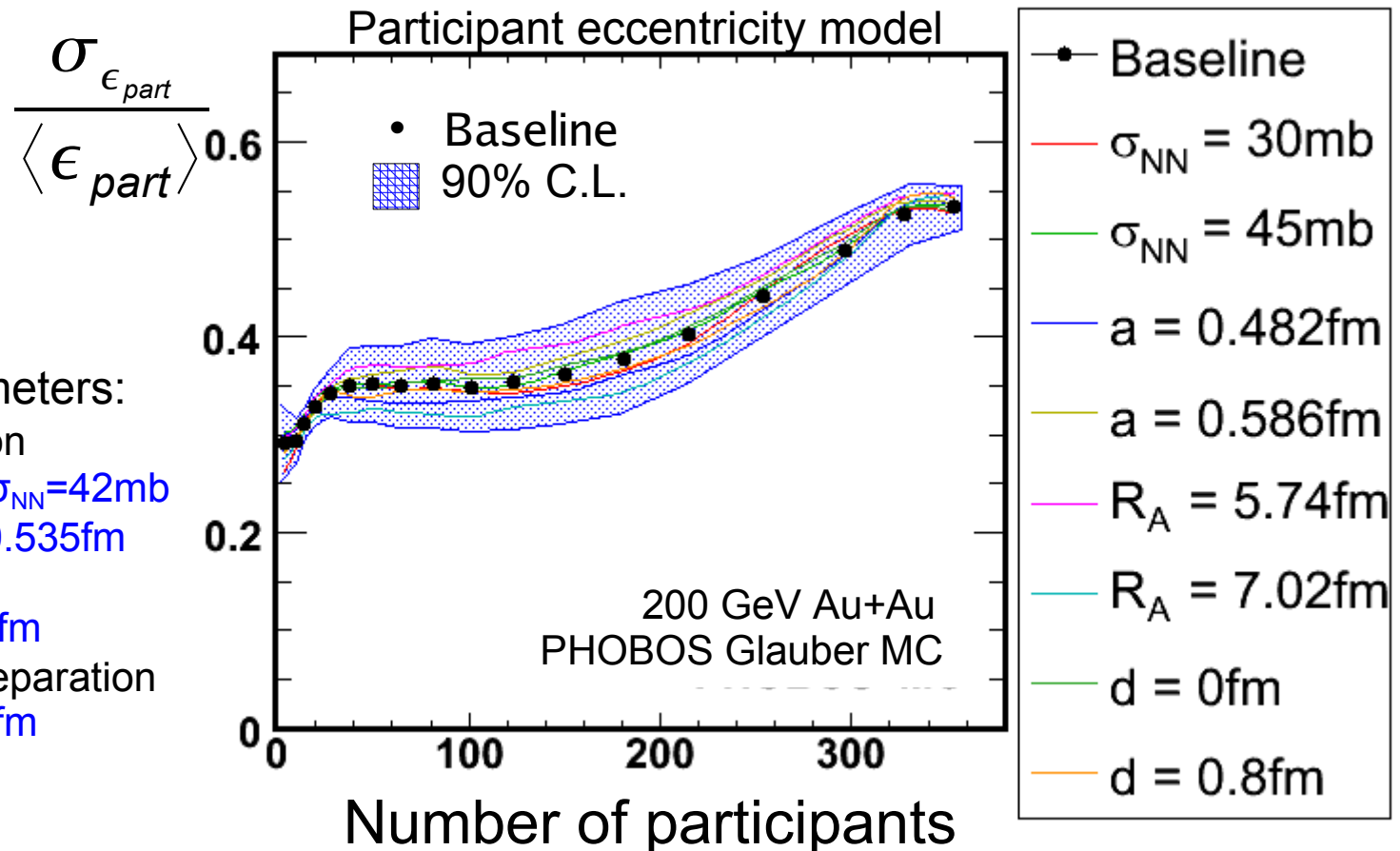
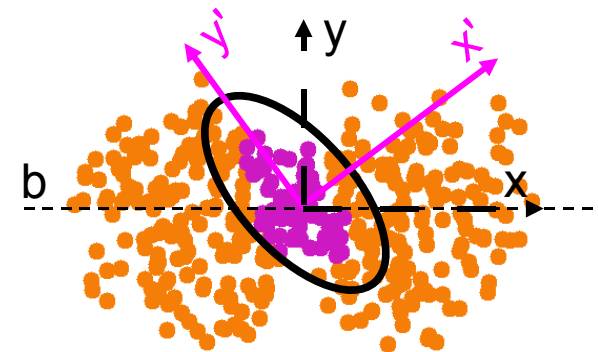
Thus, the relevant eccentricity for elliptic flow should vary event-by-event

$$\epsilon_{part} = \frac{\sqrt{(\sigma_y^2 - \sigma_x^2)^2 + 4\sigma_{xy}^2}}{\sigma_y^2 + \sigma_x^2} \quad (0 < \epsilon_{part} \leq 1)$$

Expected relative v_2 fluctuations

Elliptic flow is developed **event-by-event** with respect to the overlap region, expect

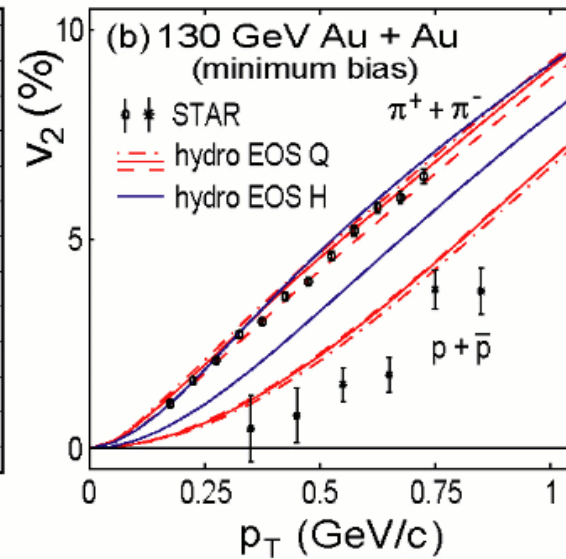
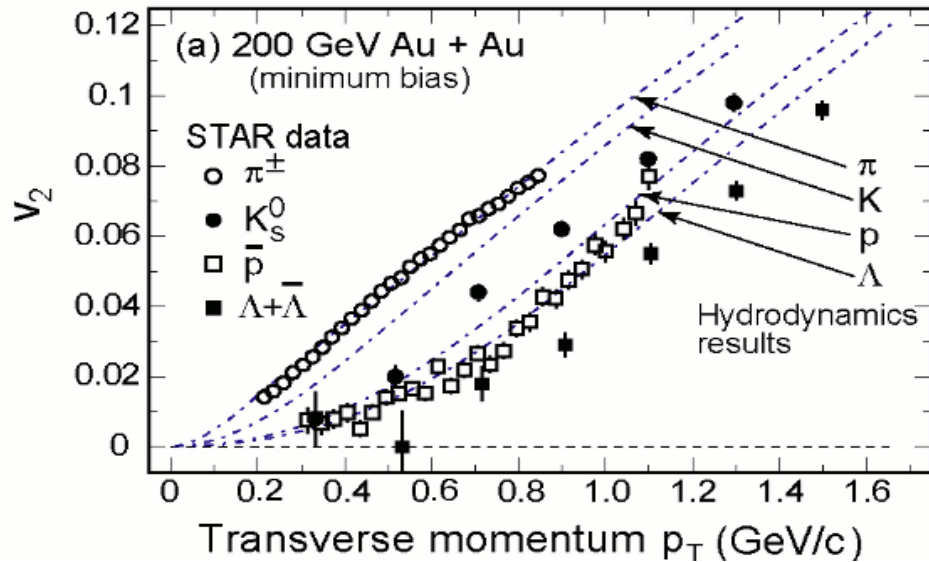
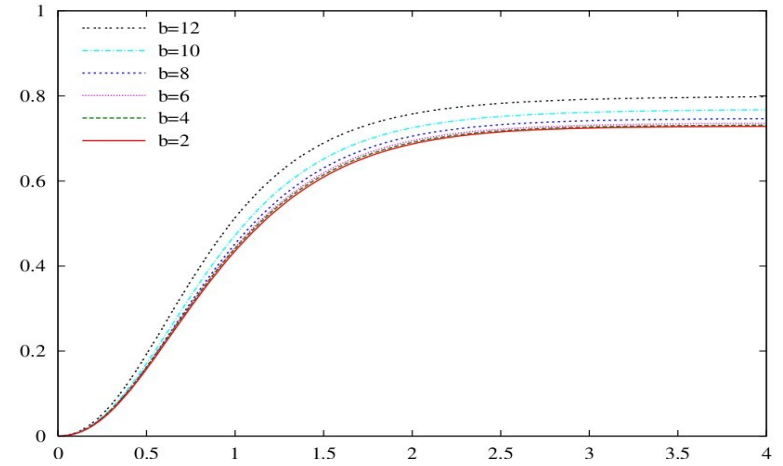
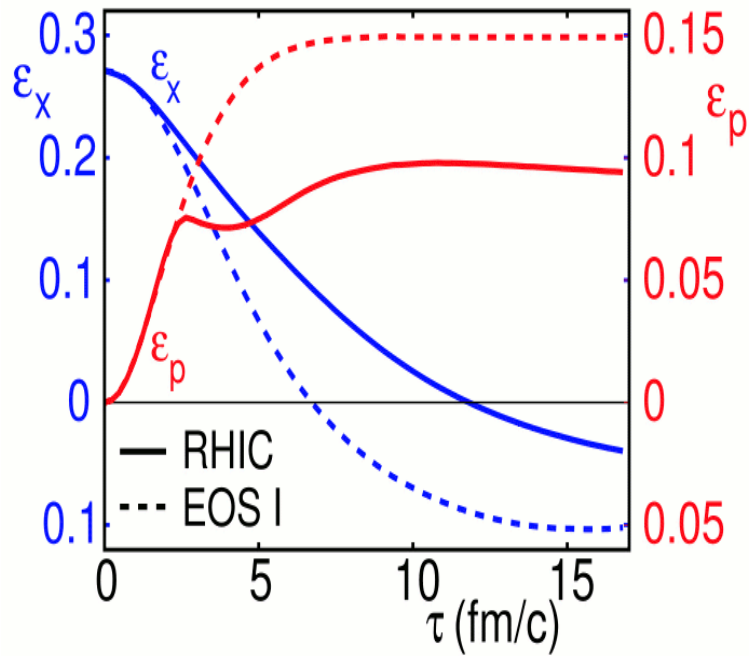
$$\frac{\sigma_{v_2}}{\langle v_2 \rangle} \sim \frac{\sigma_{\epsilon_{part}}}{\langle \epsilon_{part} \rangle}$$



Baseline parameters:

- Nucleon-nucleon cross section: $\sigma_{NN}=42\text{mb}$
- Skin depth: $a=0.535\text{fm}$
- Wood-saxon radius: $R_A=6.38\text{fm}$
- Inter-nucleon separation distance: $d=0.4\text{fm}$

Hydro features



Connection to Knudson and Reynolds number?

Define rel. flow fluctuations:

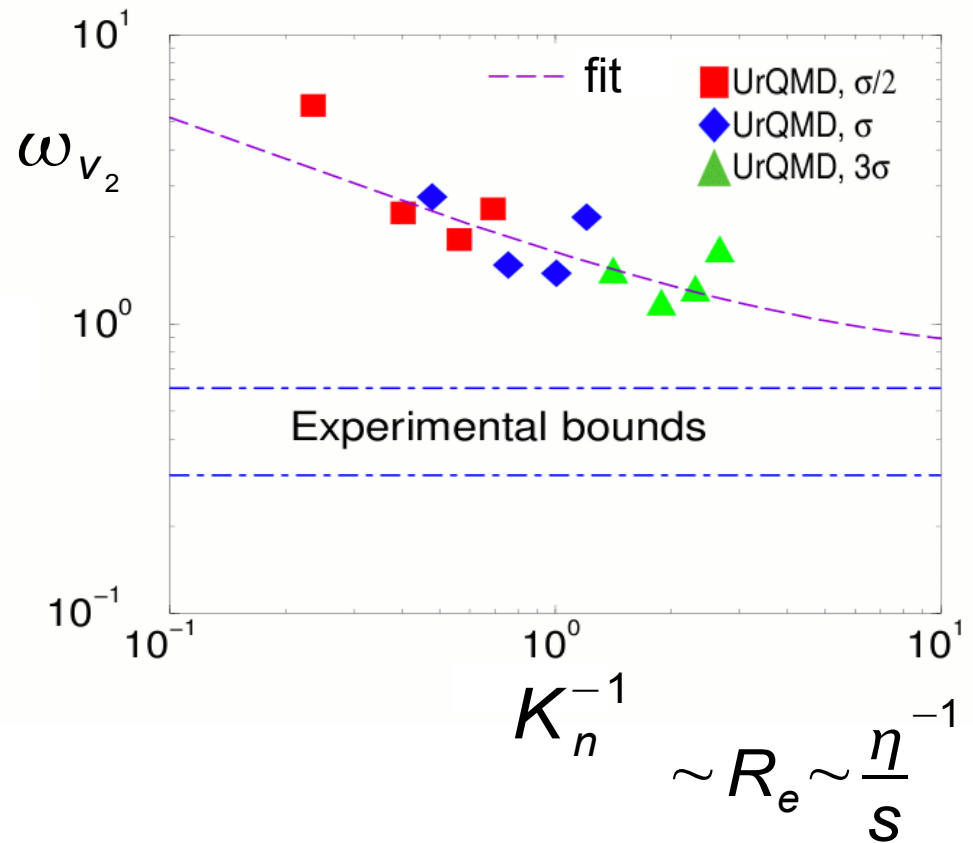
$$\omega_{v_2}^2 \equiv \frac{\sigma_{v_2}^2}{\langle v_2 \rangle^2} = \frac{\sigma_{\epsilon_{part}}^2}{\langle \epsilon_{part} \rangle^2} + \Delta_{dyn}^2$$

Define the inverse of the Knudson, the average number of collisions suffered by a dof in the system:

$$K_n^{-1} = L/\lambda$$

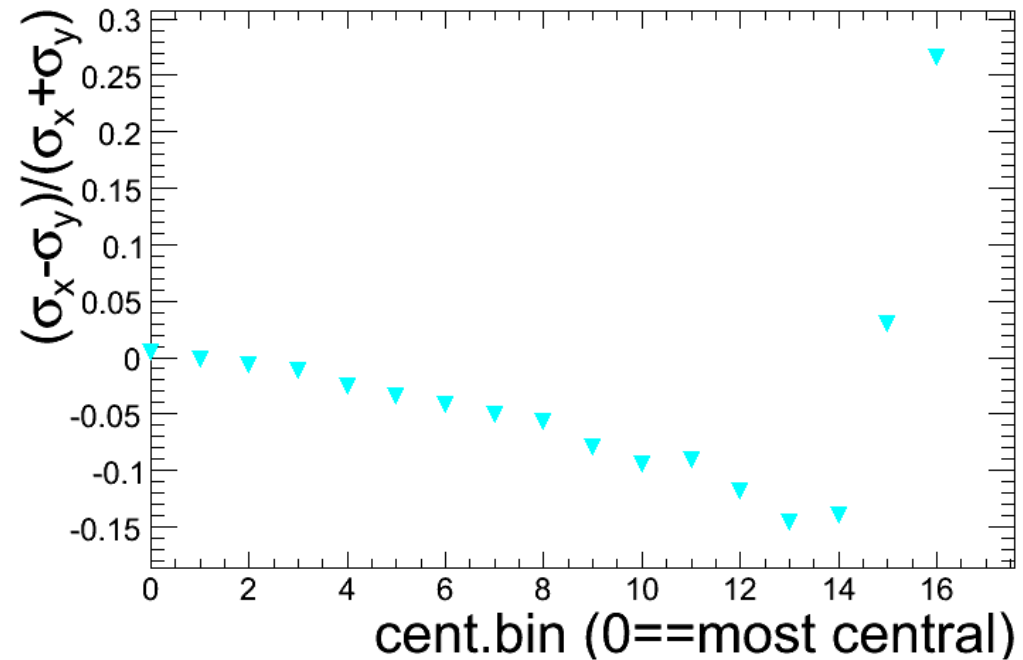
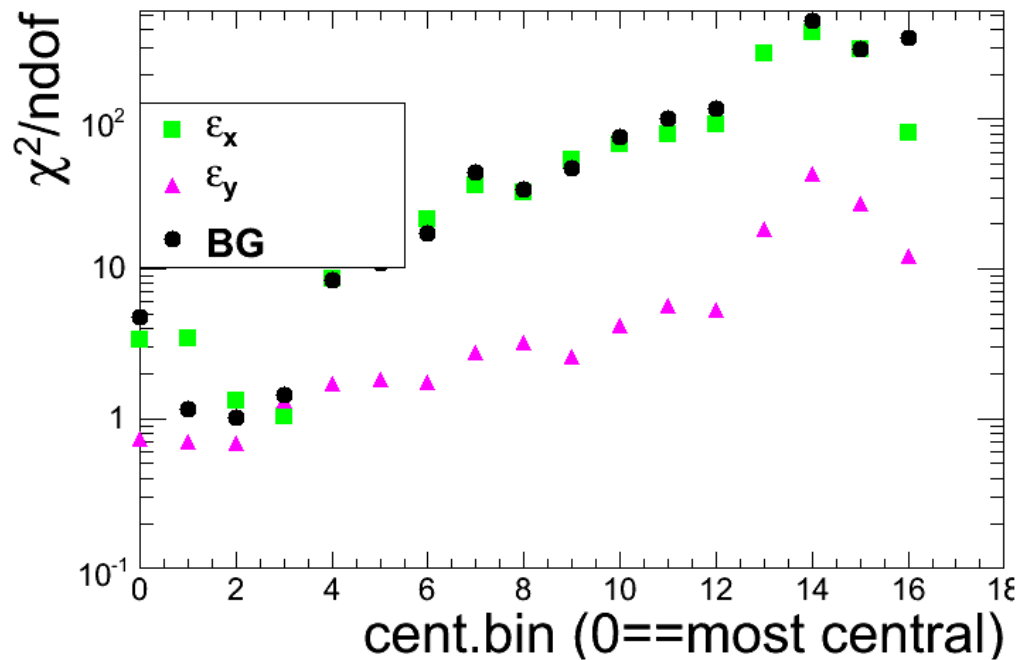
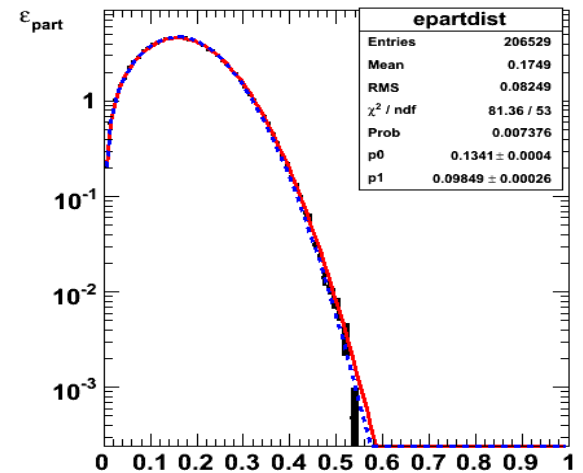
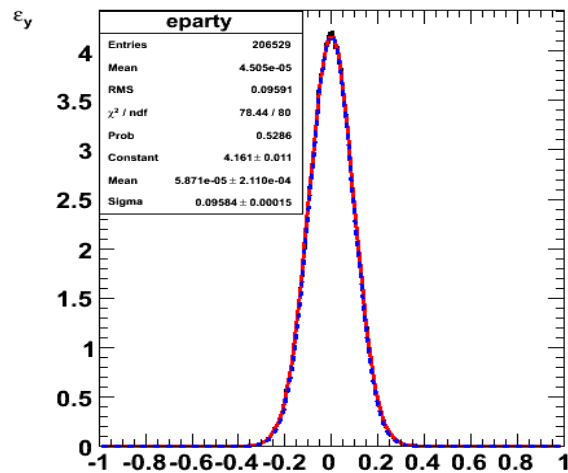
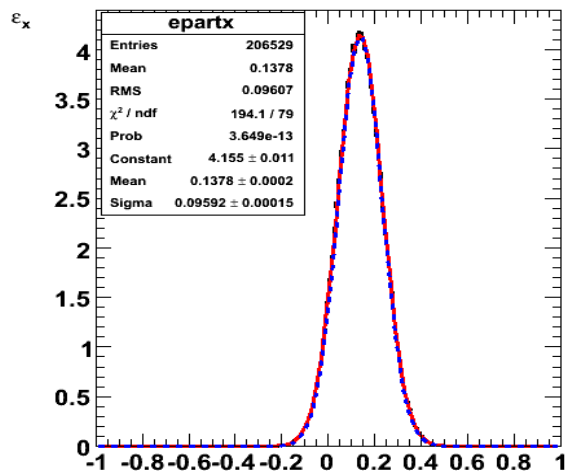
Assume Poissonian:

$$\Delta_{dyn} \sim \alpha \sqrt{K_n}$$

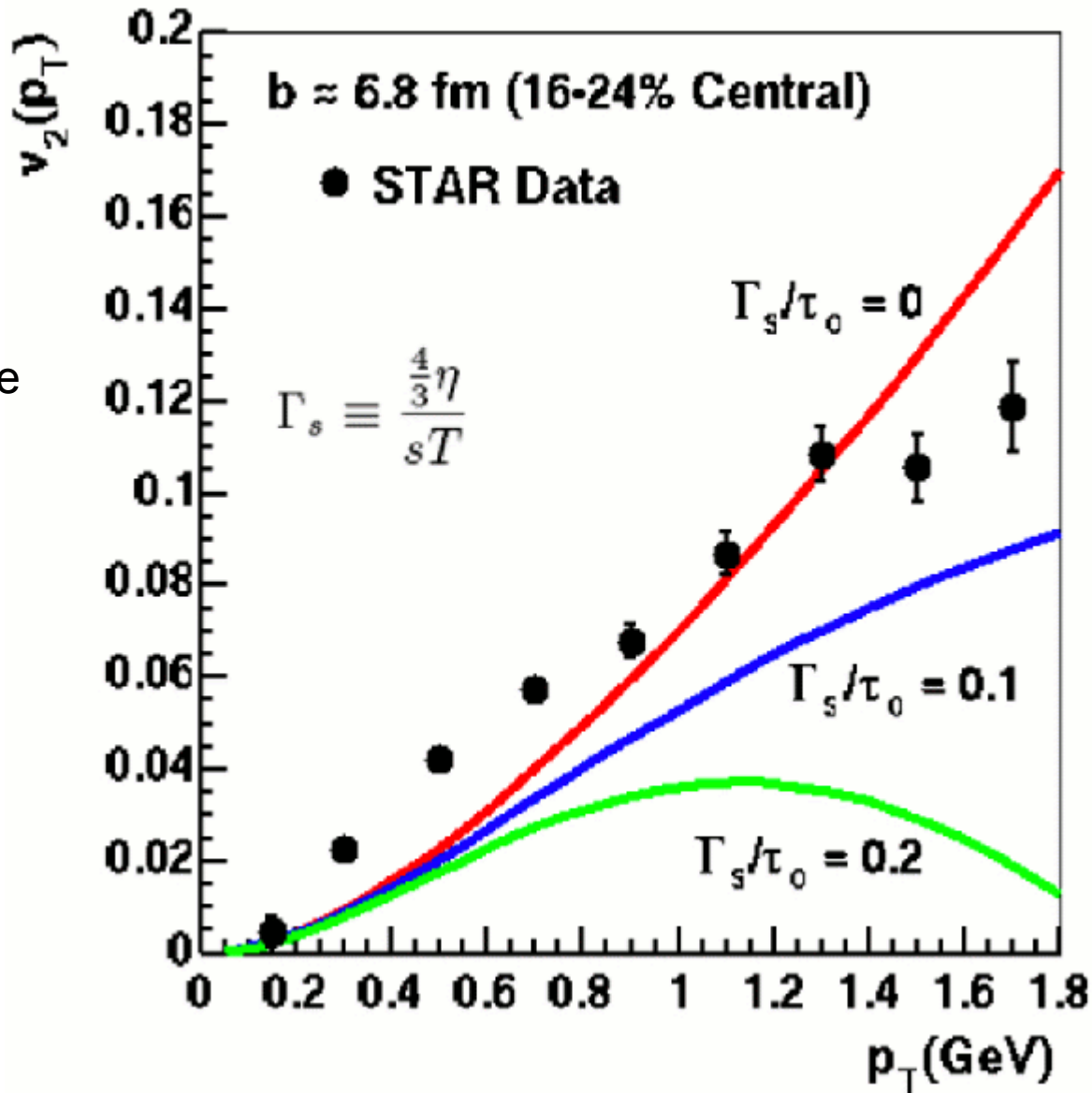


Conclusion/speculation(?) that viscosity must be large enough to avoid strong turbulences (that are not seen in the data)

Gaussian flow fluctuations



Viscous corrections



Sound attenuation length is approx the same as the mean free path